Optimal Any-angle Pathfinding

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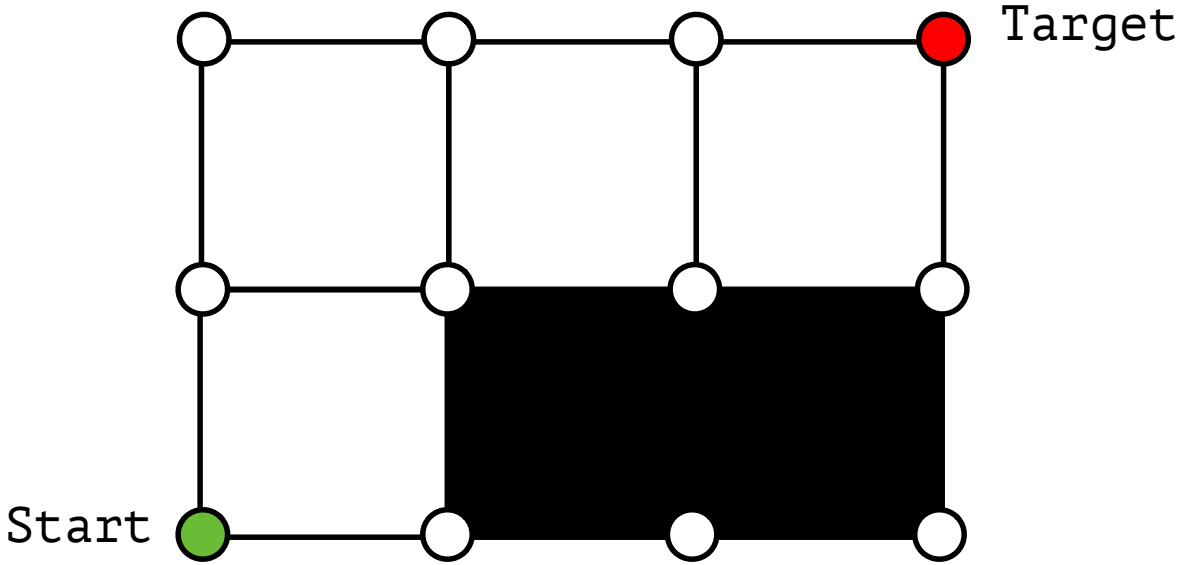
Joint work with:
Vural Aksakalli, Michael Cui, Alban Grastien and Dindar Öz

AAMAS Tutorial
2019-05-14

Suboptimality and Theta*

Theta* [Nash *et al.*, 2007] proceeds from one grid vertex to the next. This strategy is suboptimal since it expands nodes out of order.



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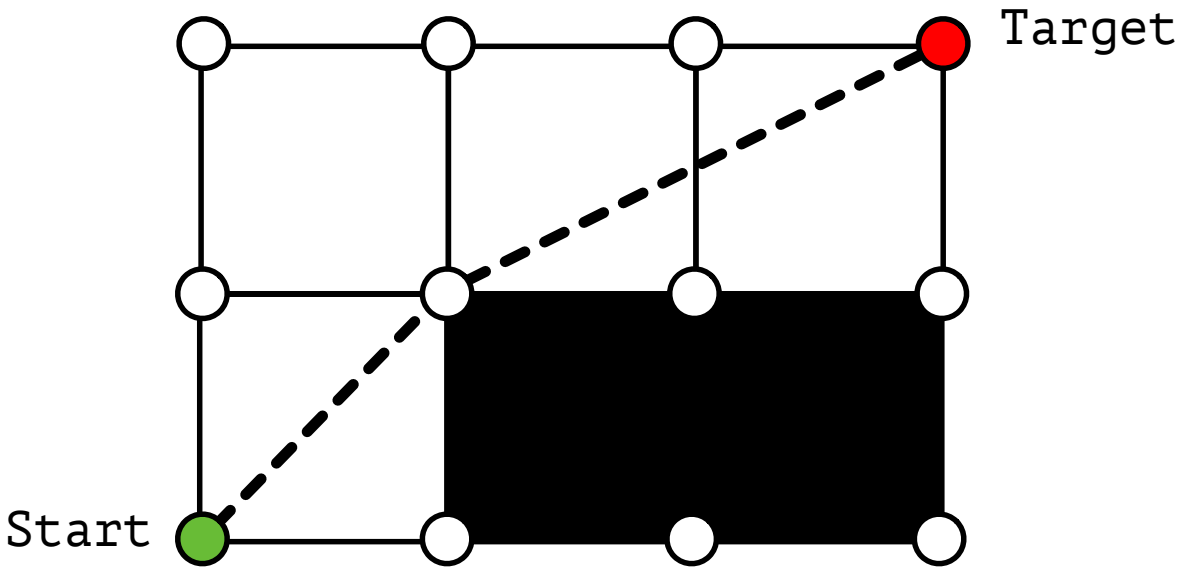

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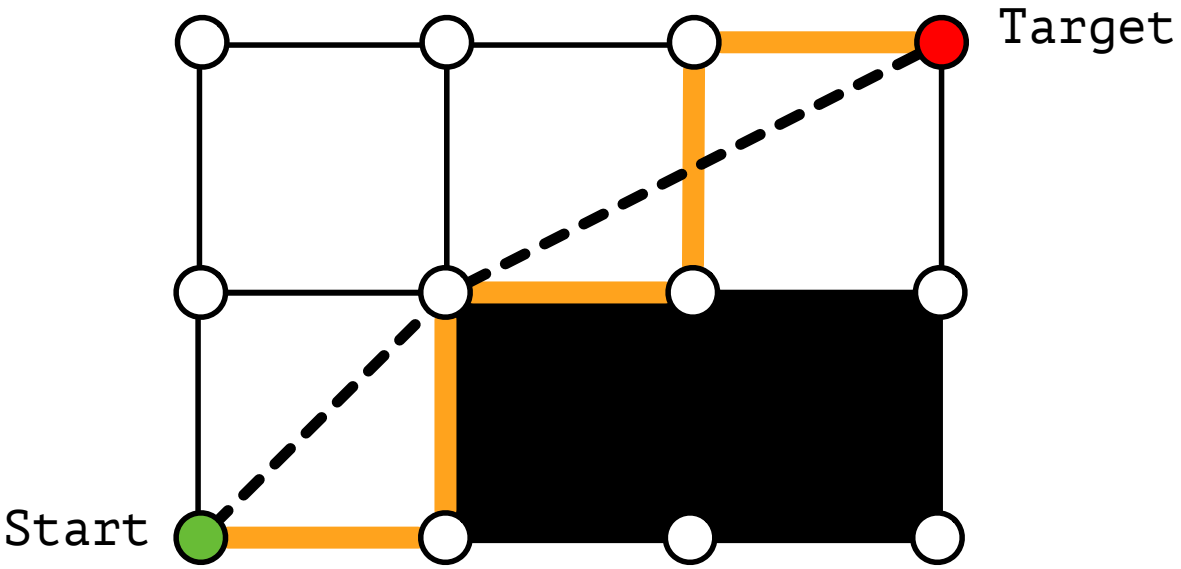

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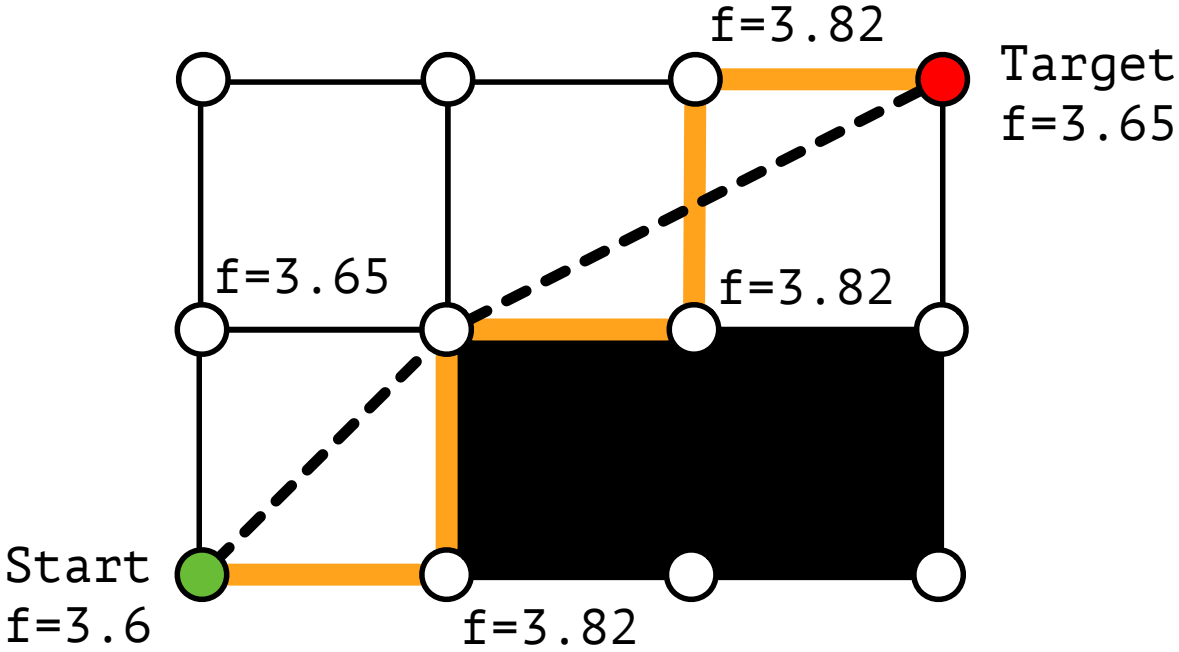

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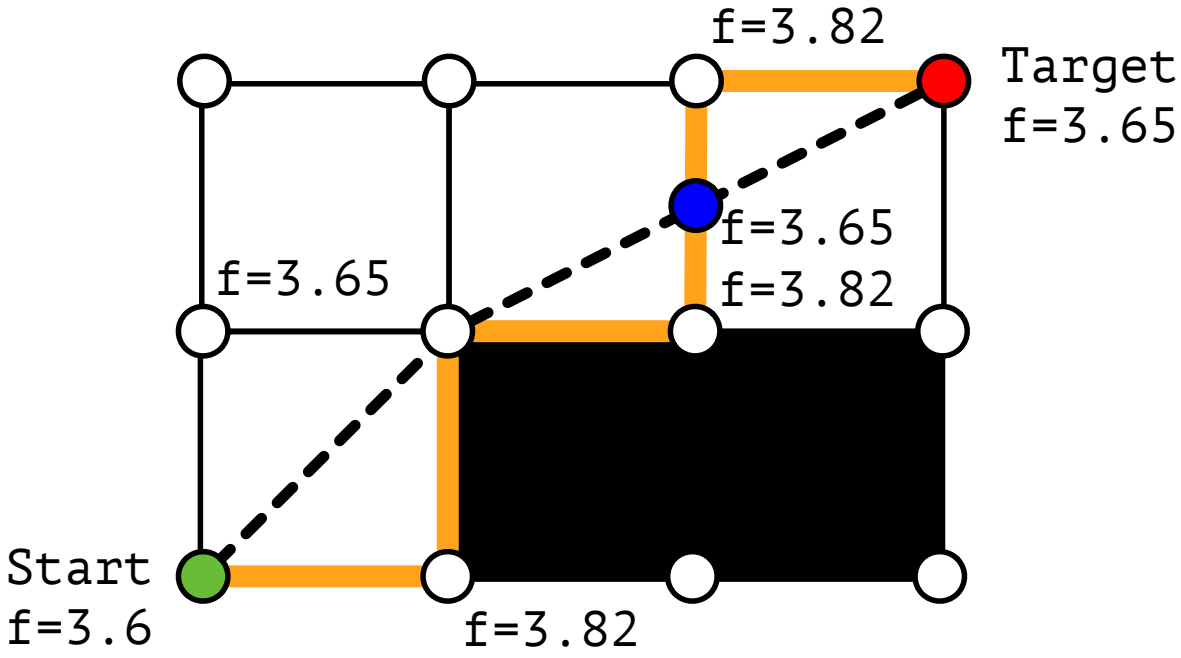
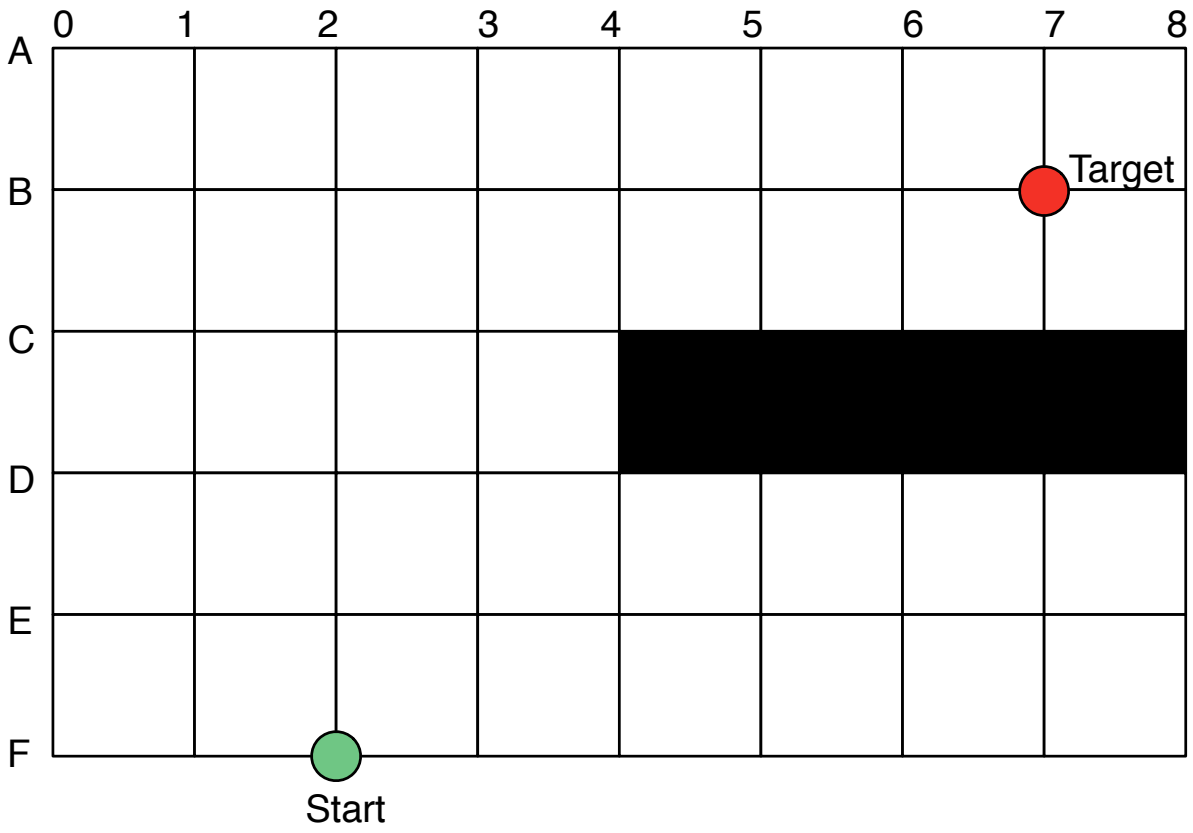


Anya: An optimal any-angle pathfinding technique

Intuition

Expand *sets* of nodes together at one time [Harabor *et al.*, 2016]. A set is a contiguous intervals of points from along a single row.



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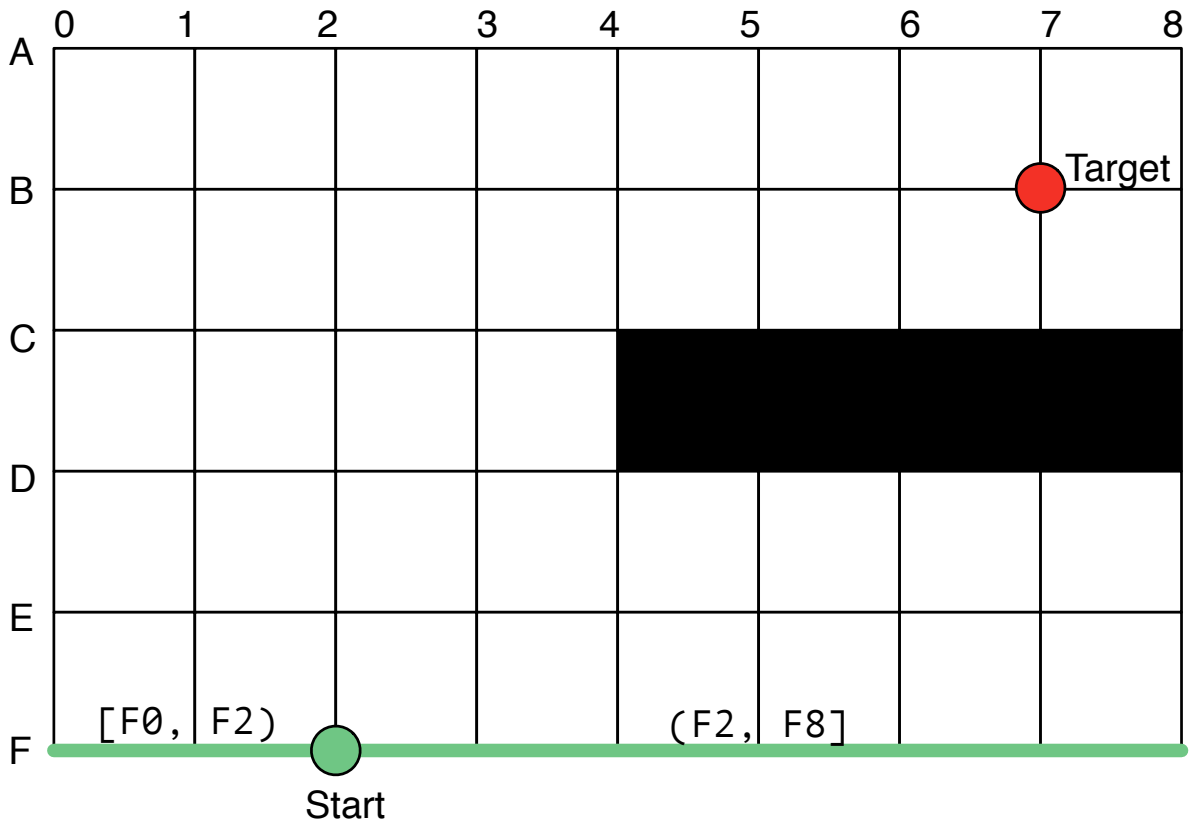

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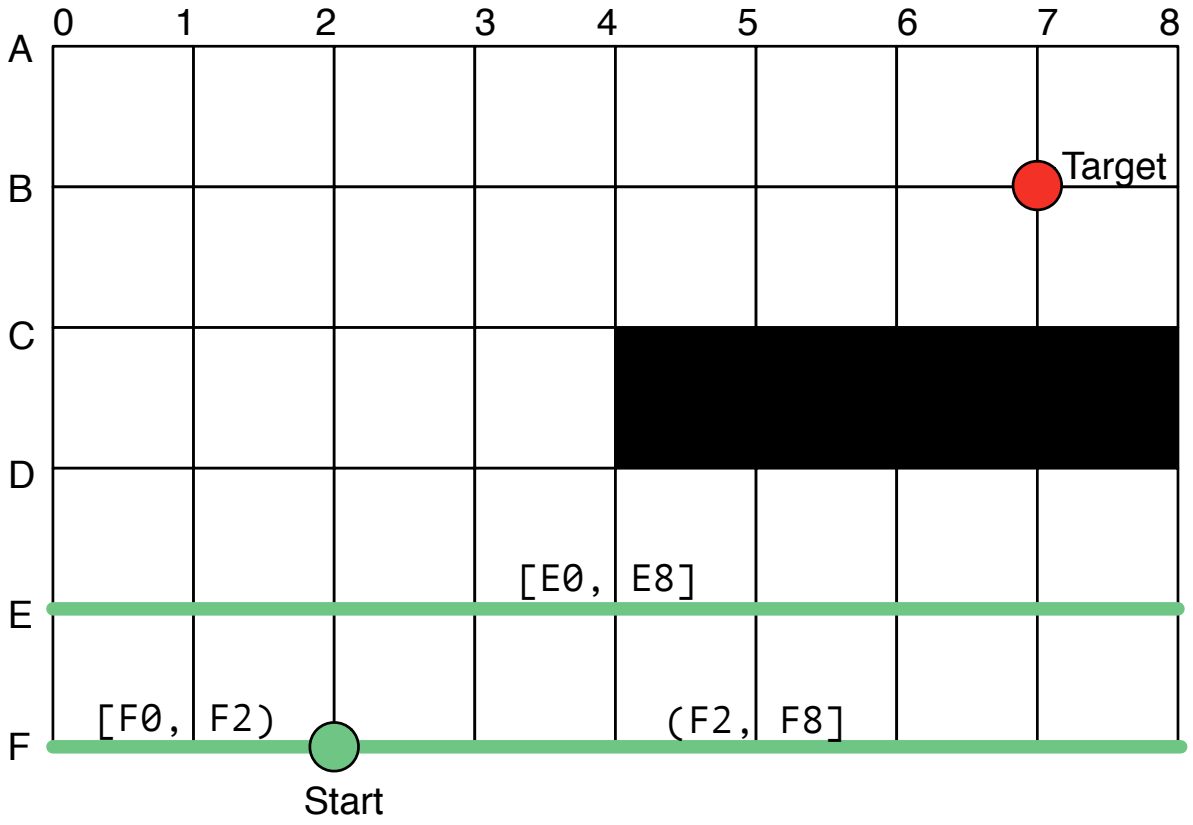

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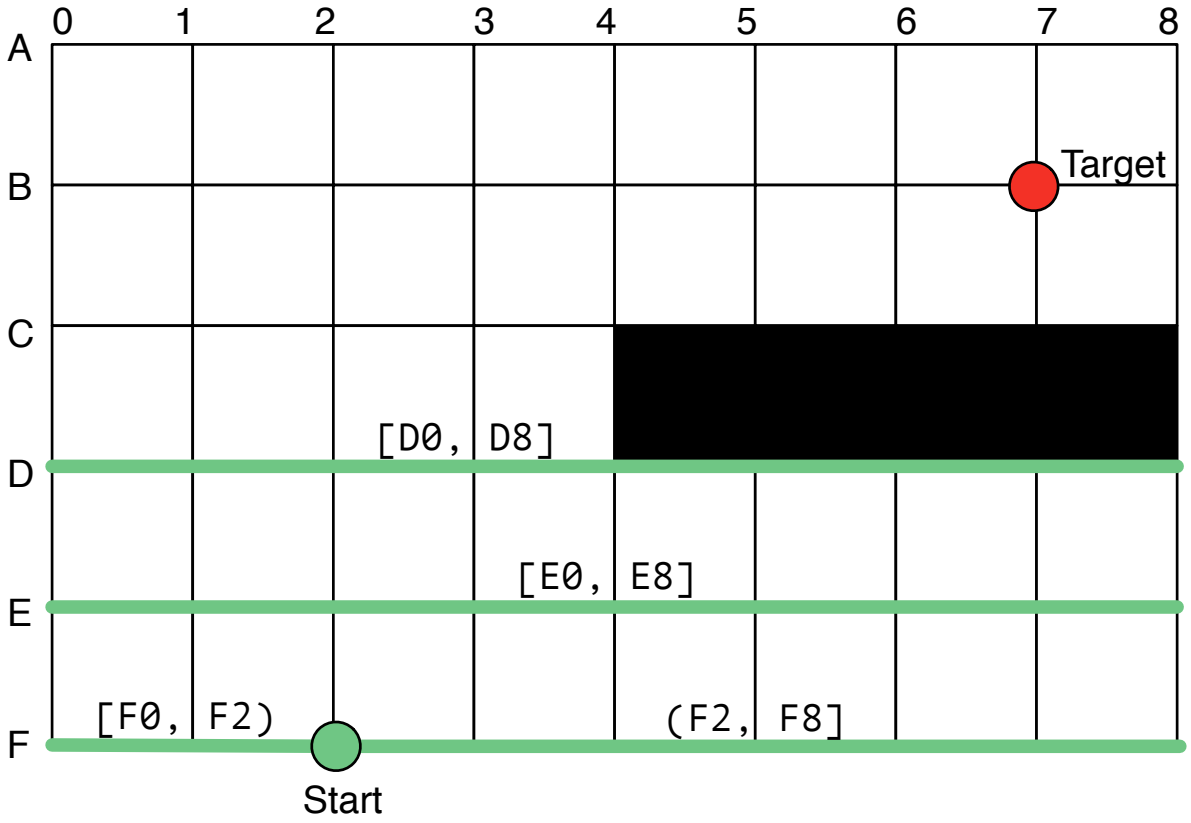

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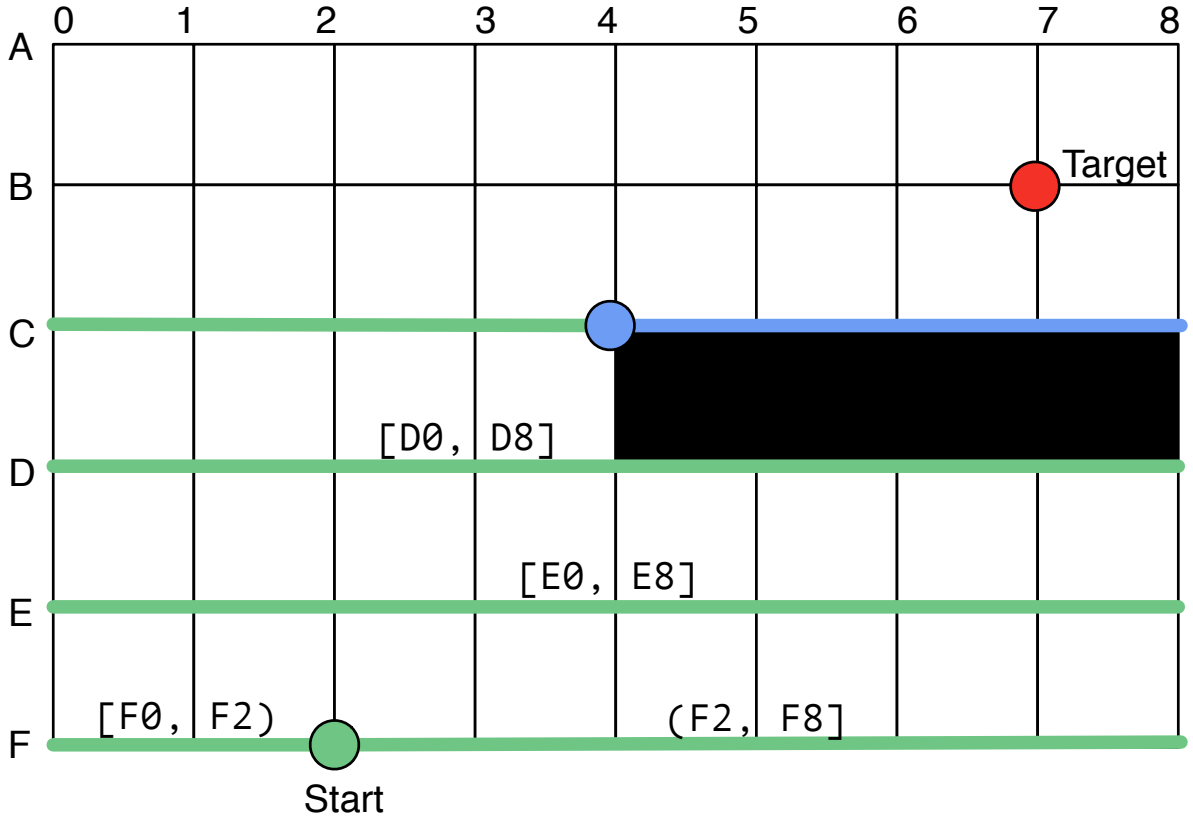

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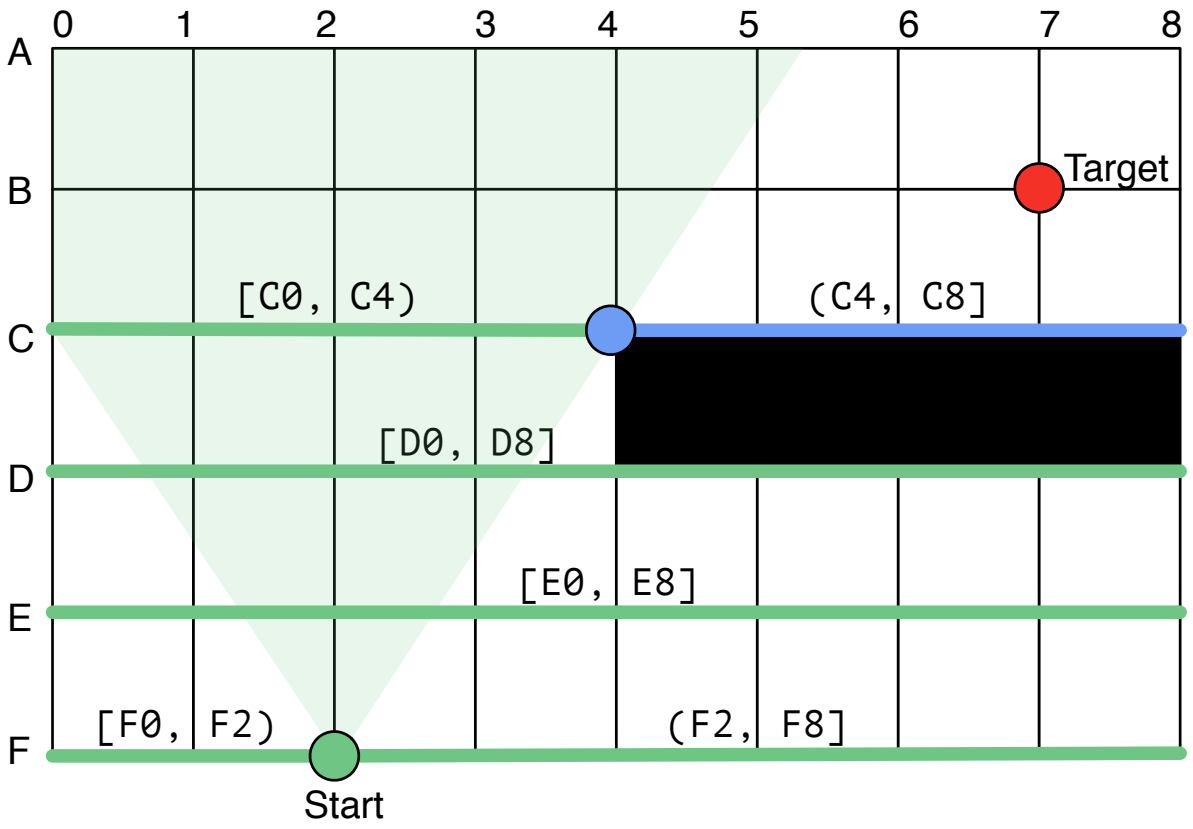
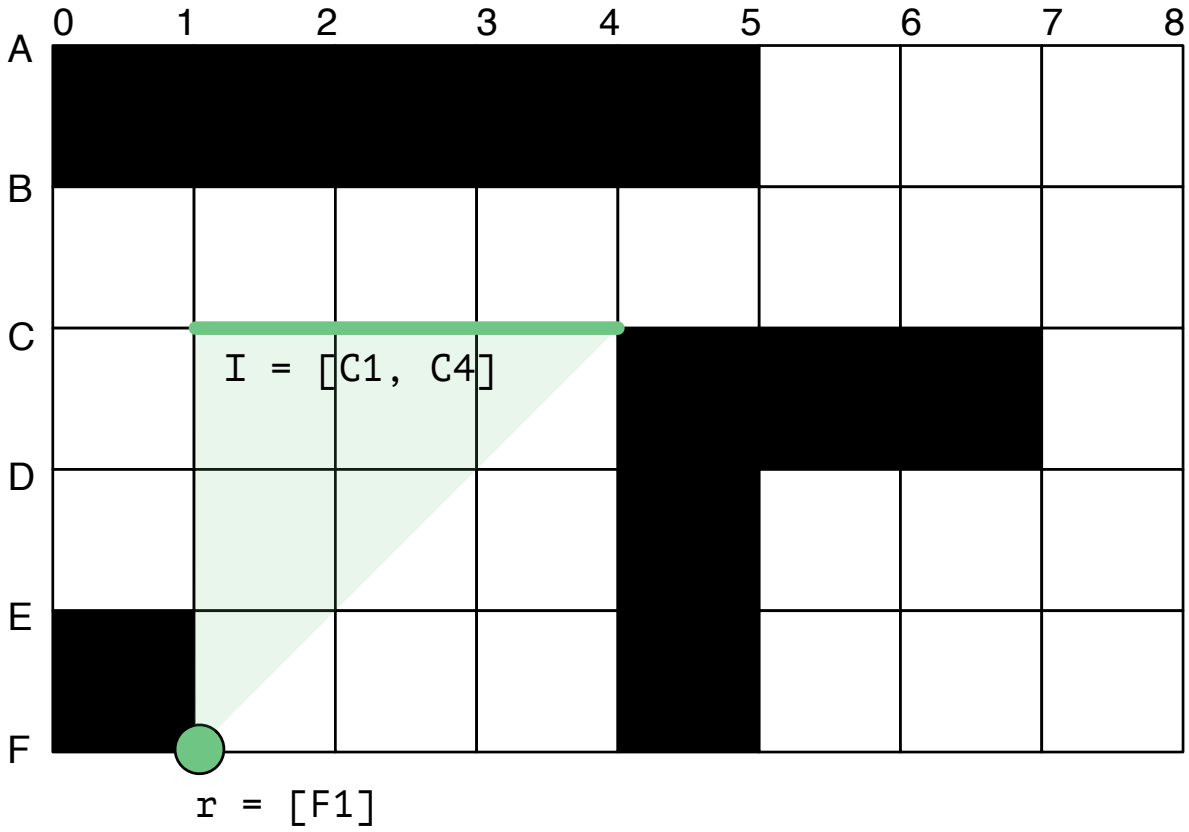


Definition #1: Search Nodes

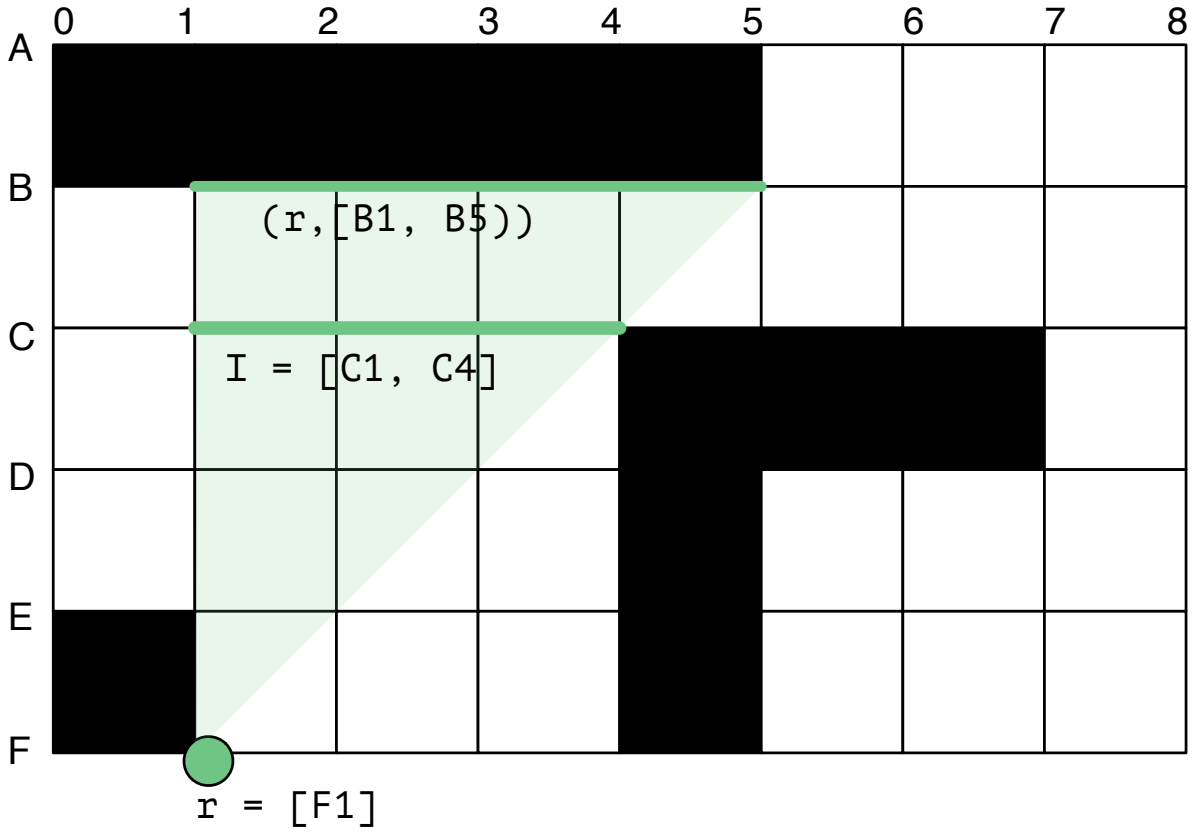
Every node is a tuple (I, r) where:

- r is a *root*; the most recent turning point.
- I is an interval of contiguous points, all visible from r.
- The *start node* has a point interval and a root "off the grid"



Definition #2: Successors

- Successors of node (I, r) are found by traveling from r and through I along a locally taut path.
- Two kinds of successors: *observable* and *non-observable*



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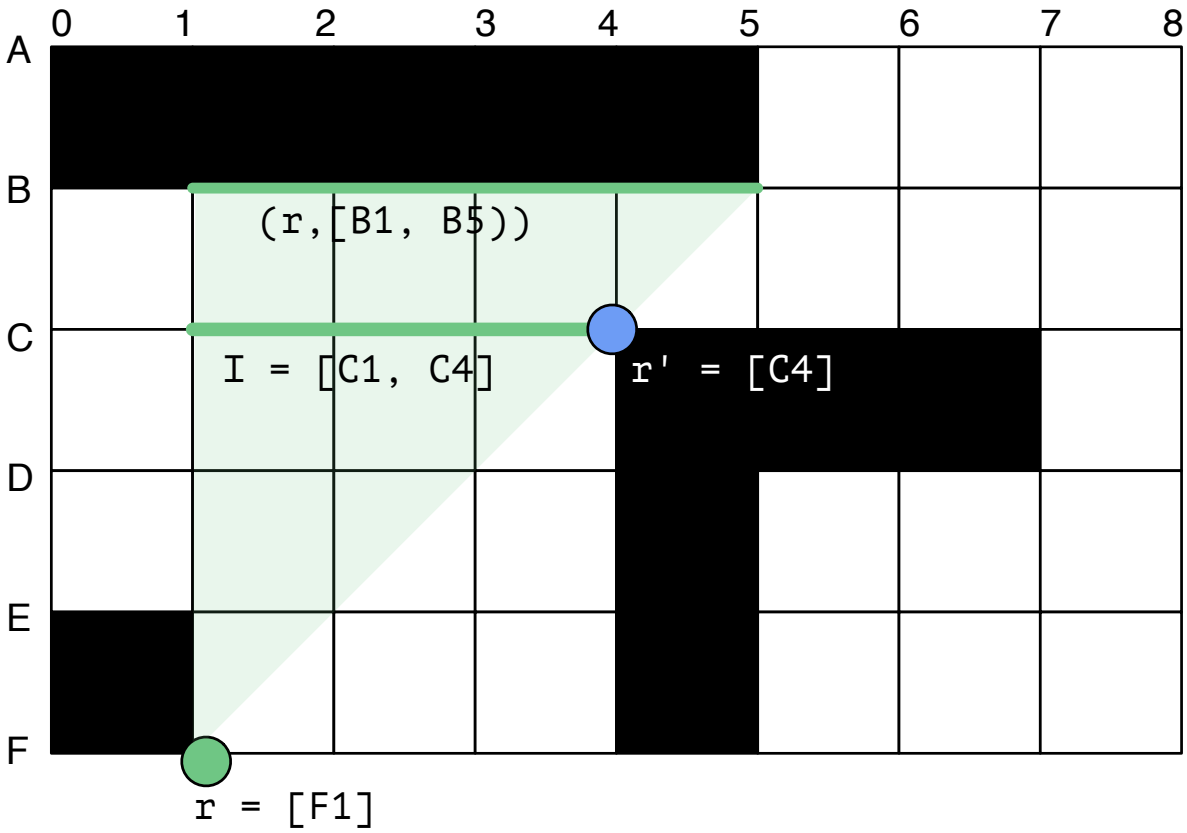

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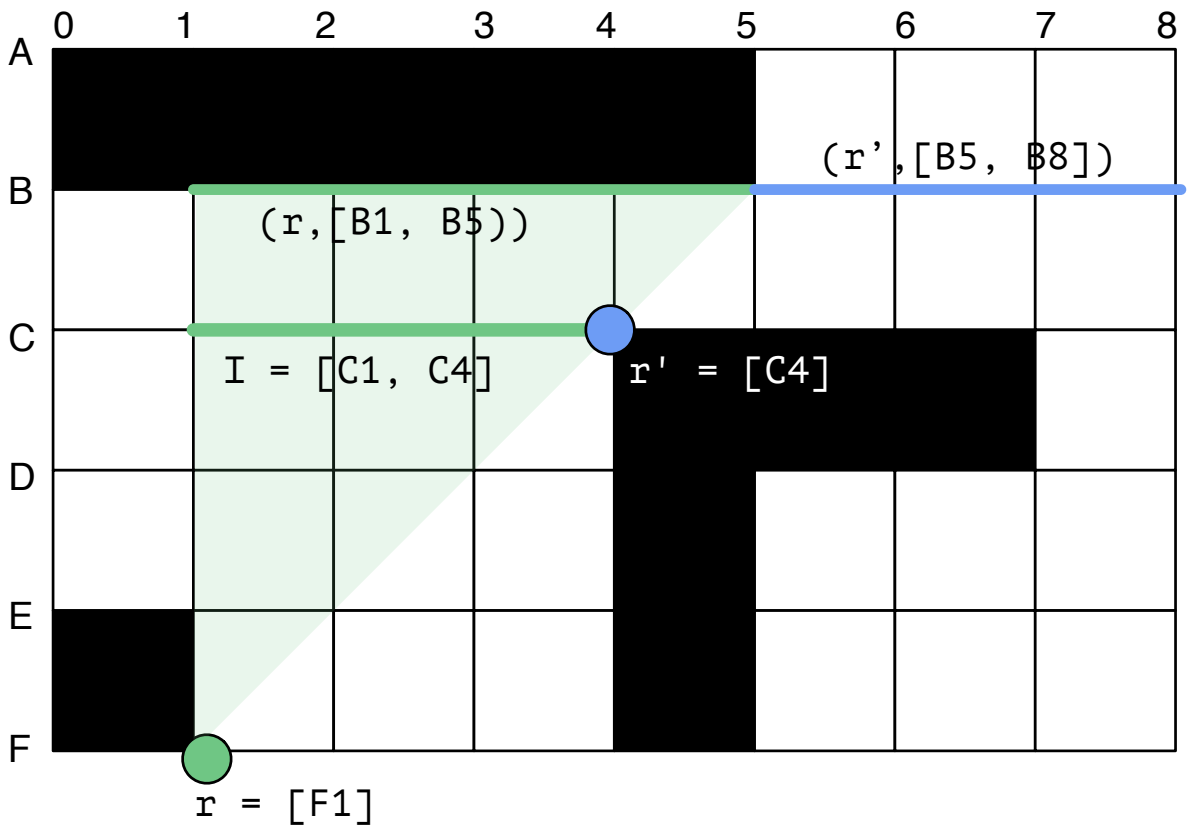

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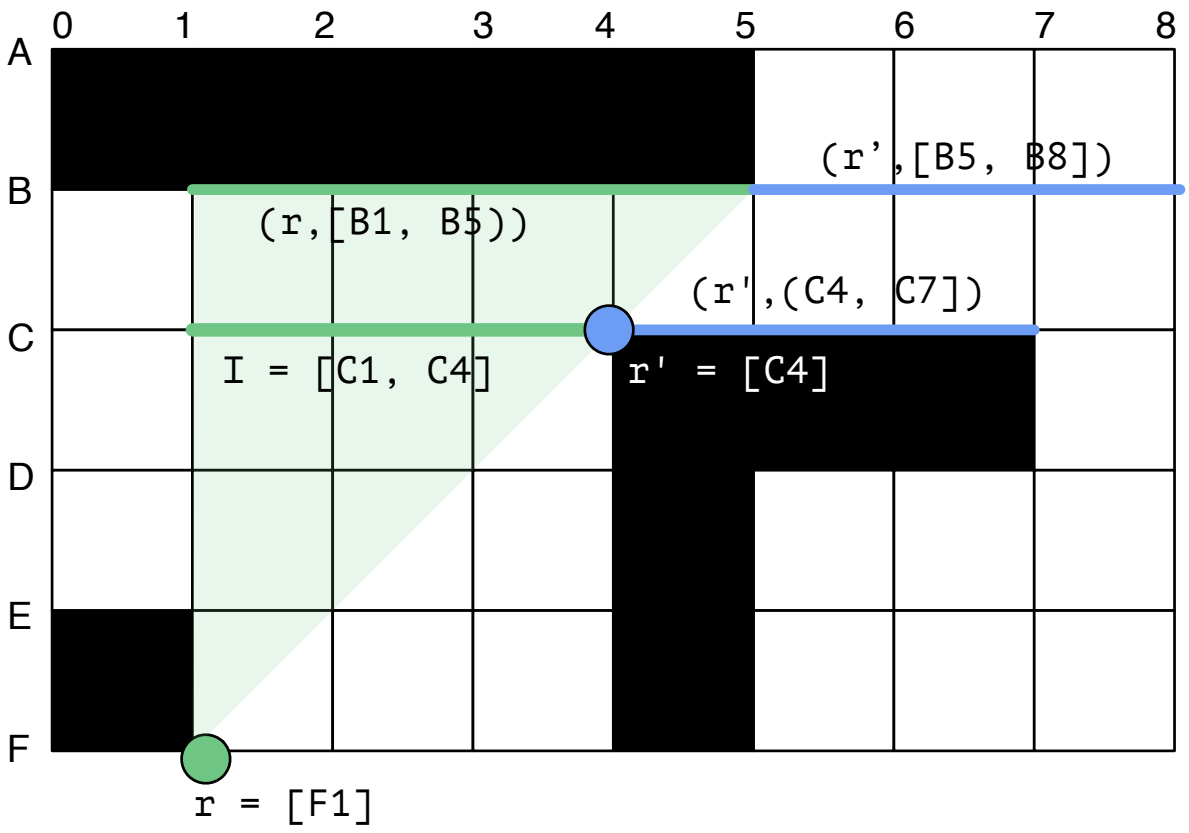
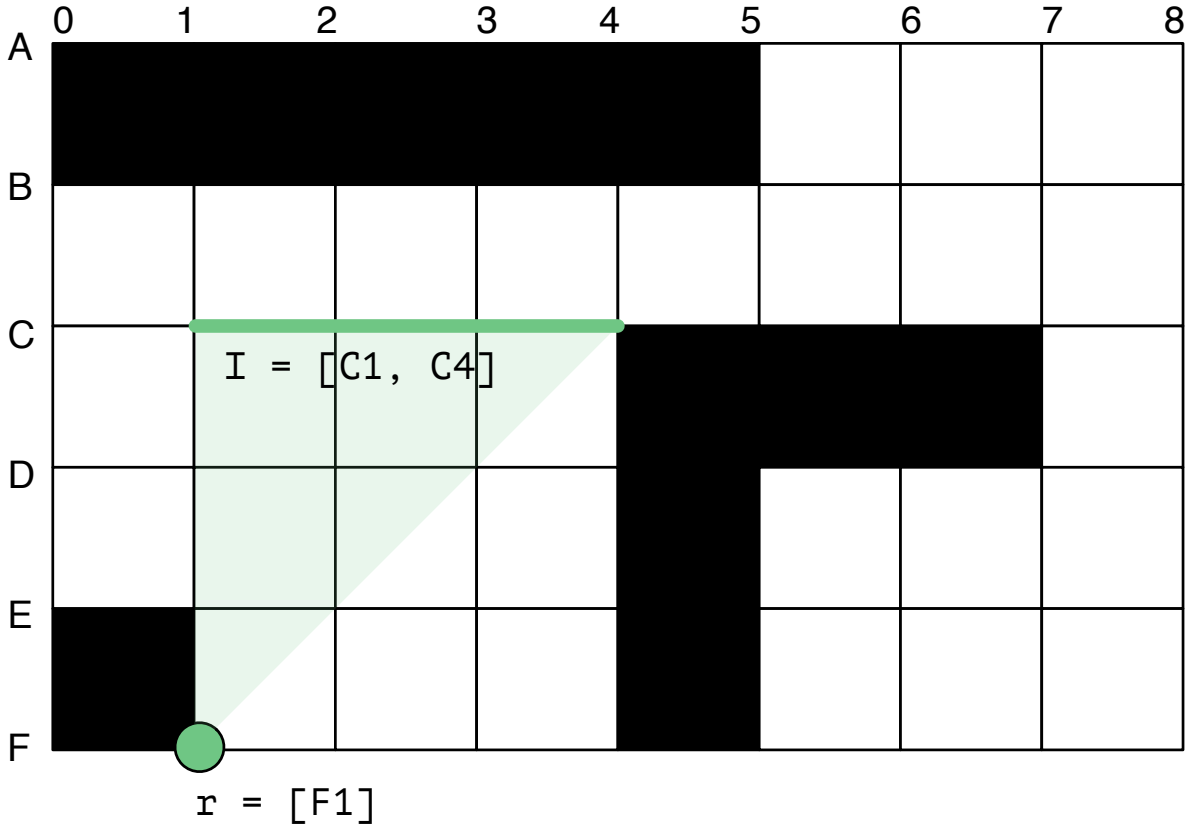


Evaluation Function

- From each interval I we choose a single point p which minimises the cost-to-go (i.e. the f-value of the node).



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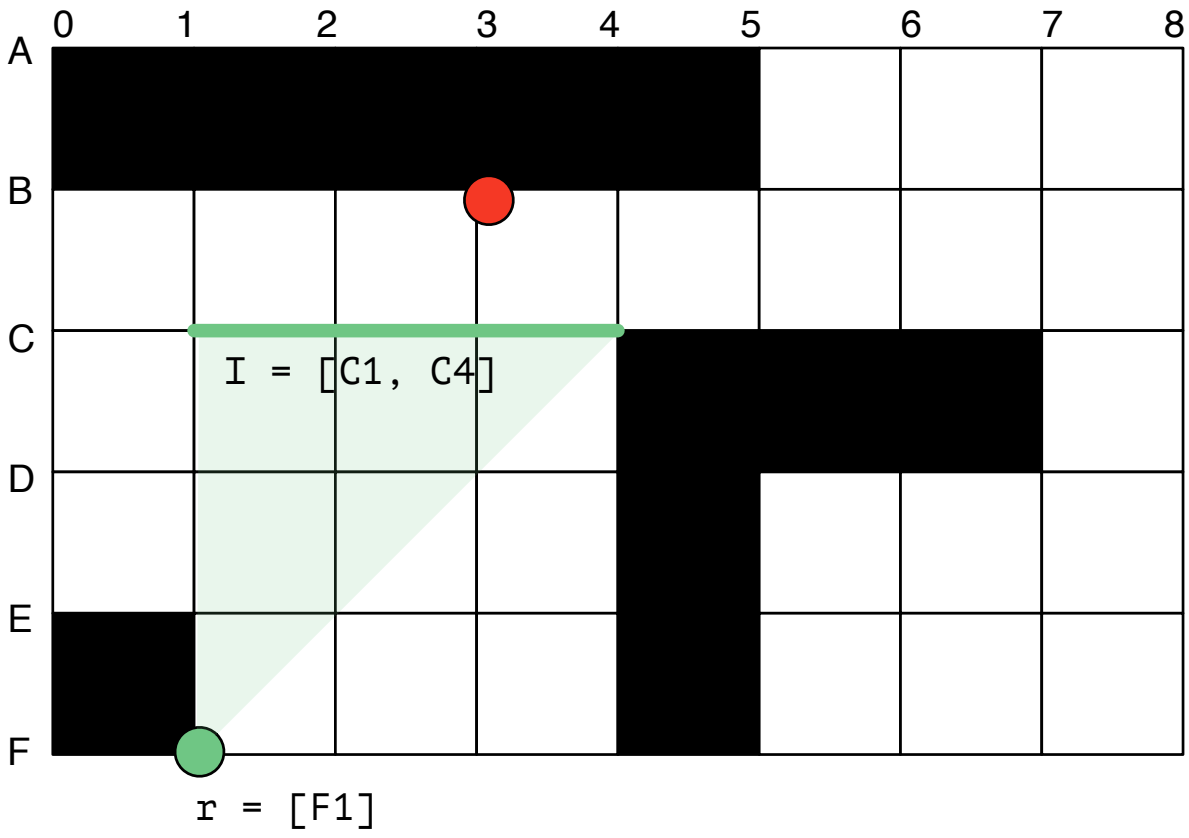

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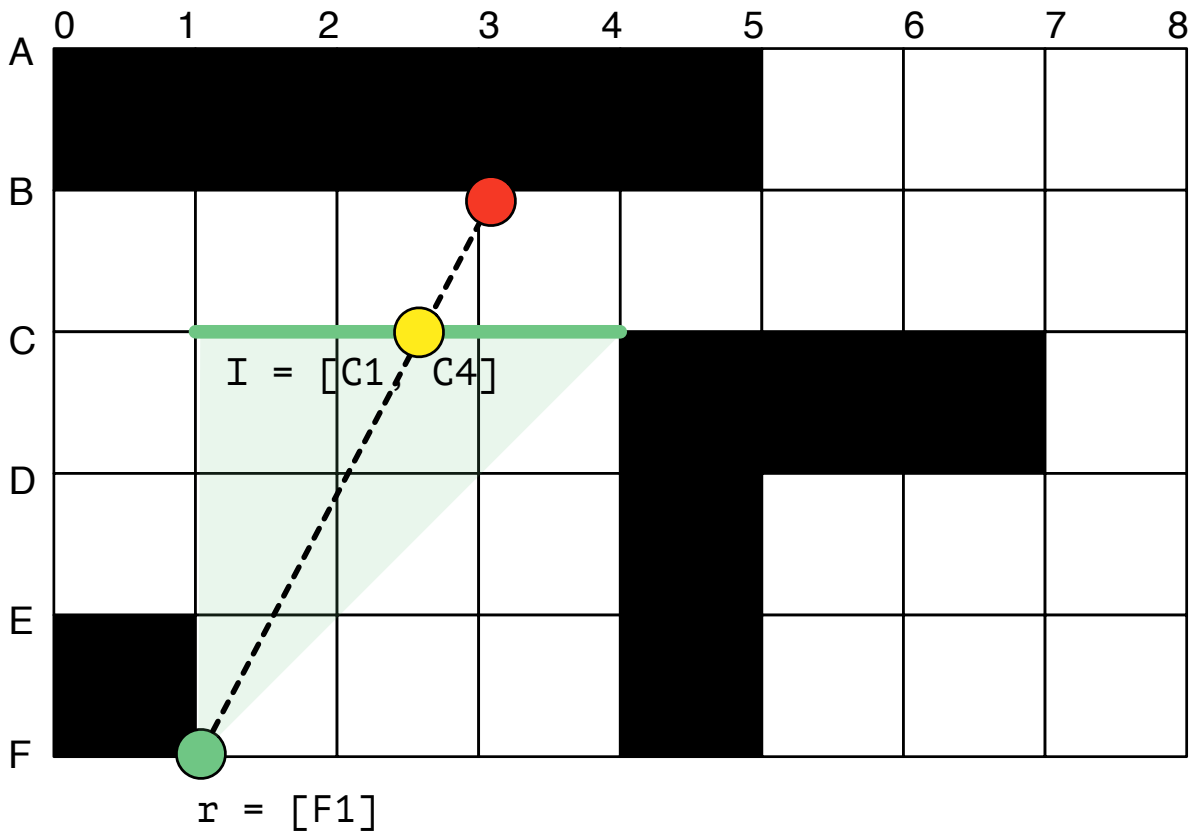

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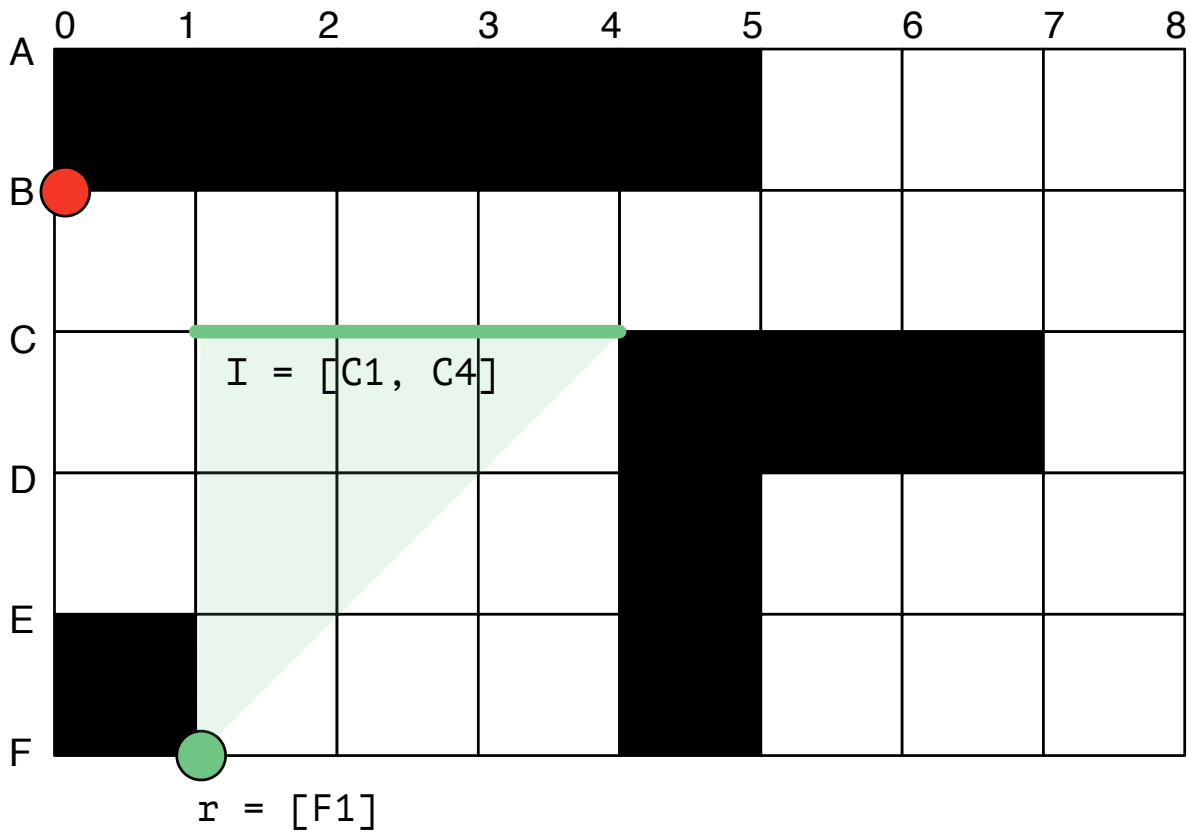

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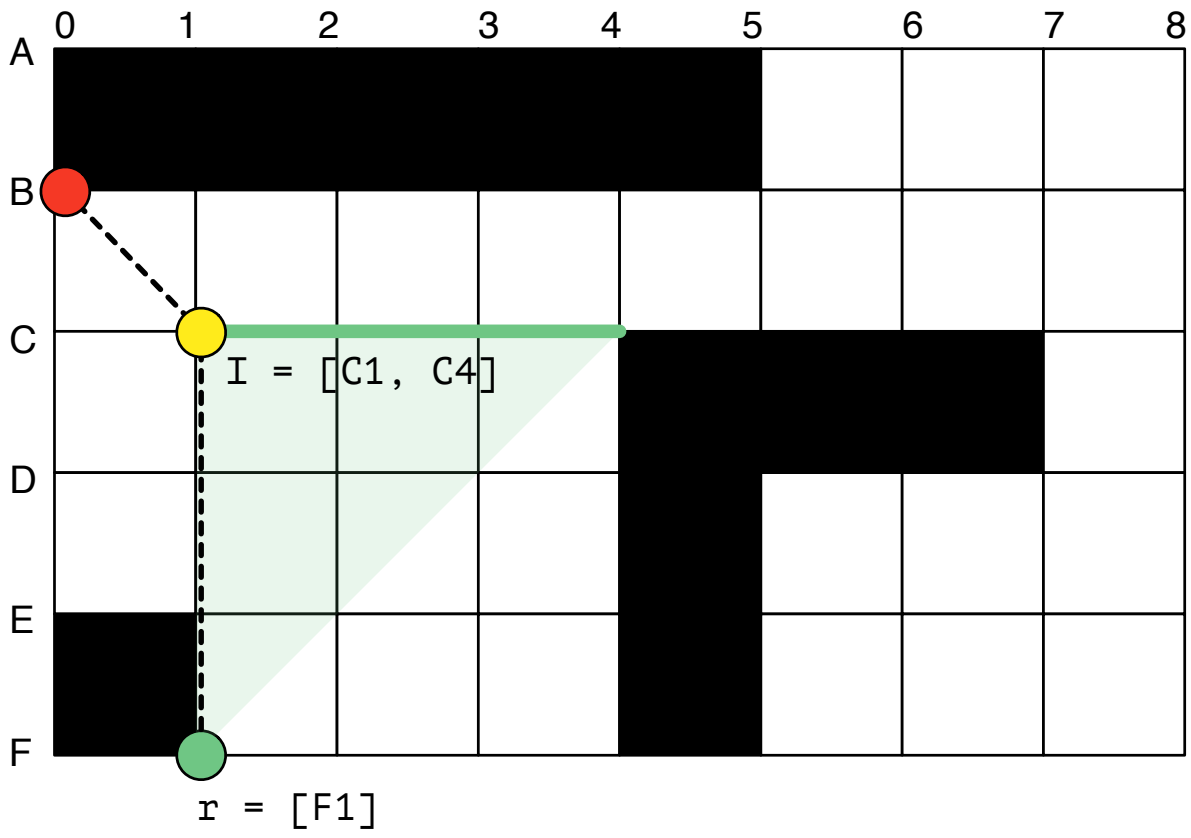

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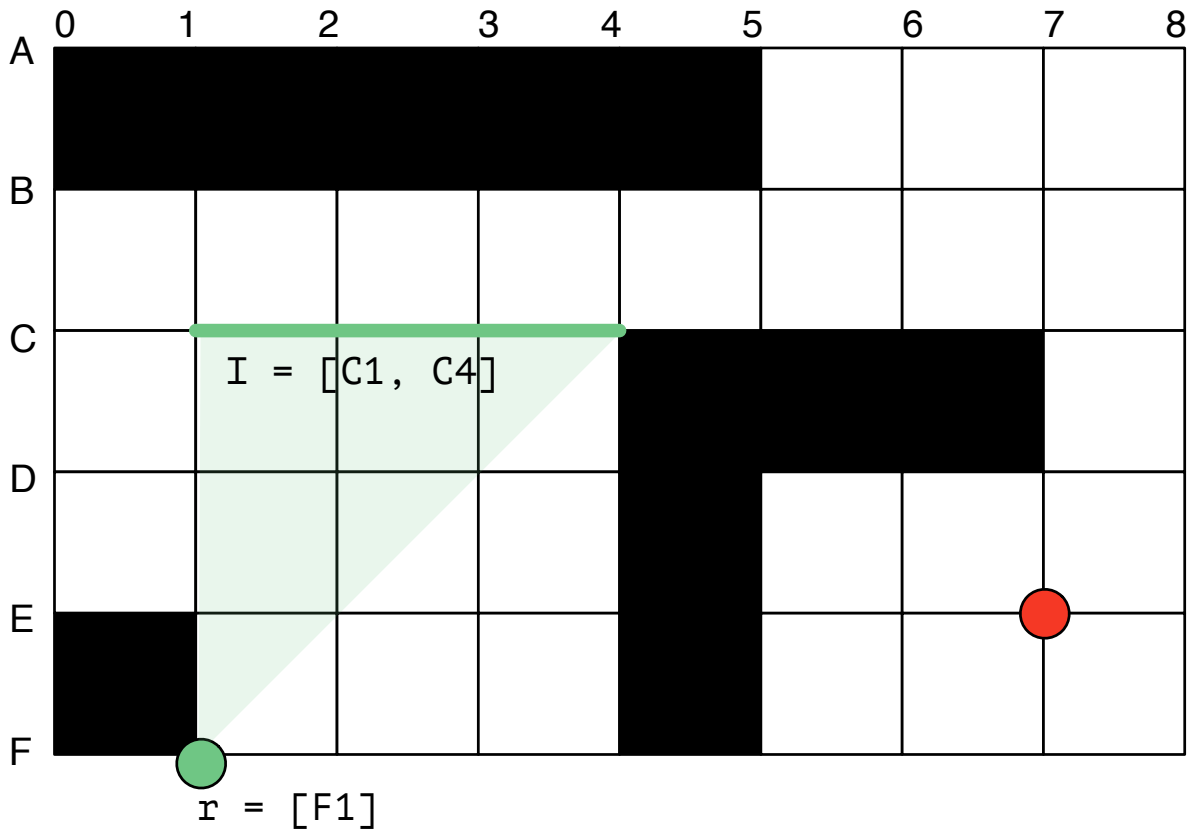

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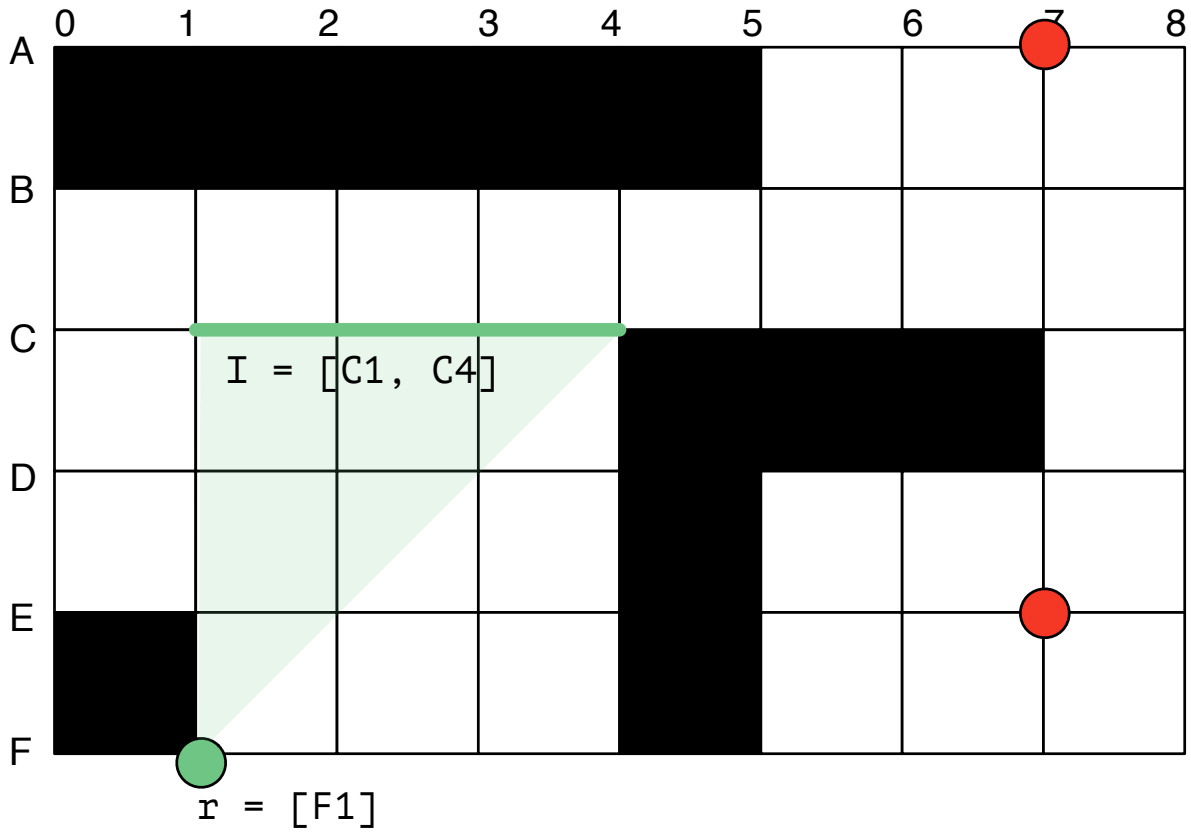

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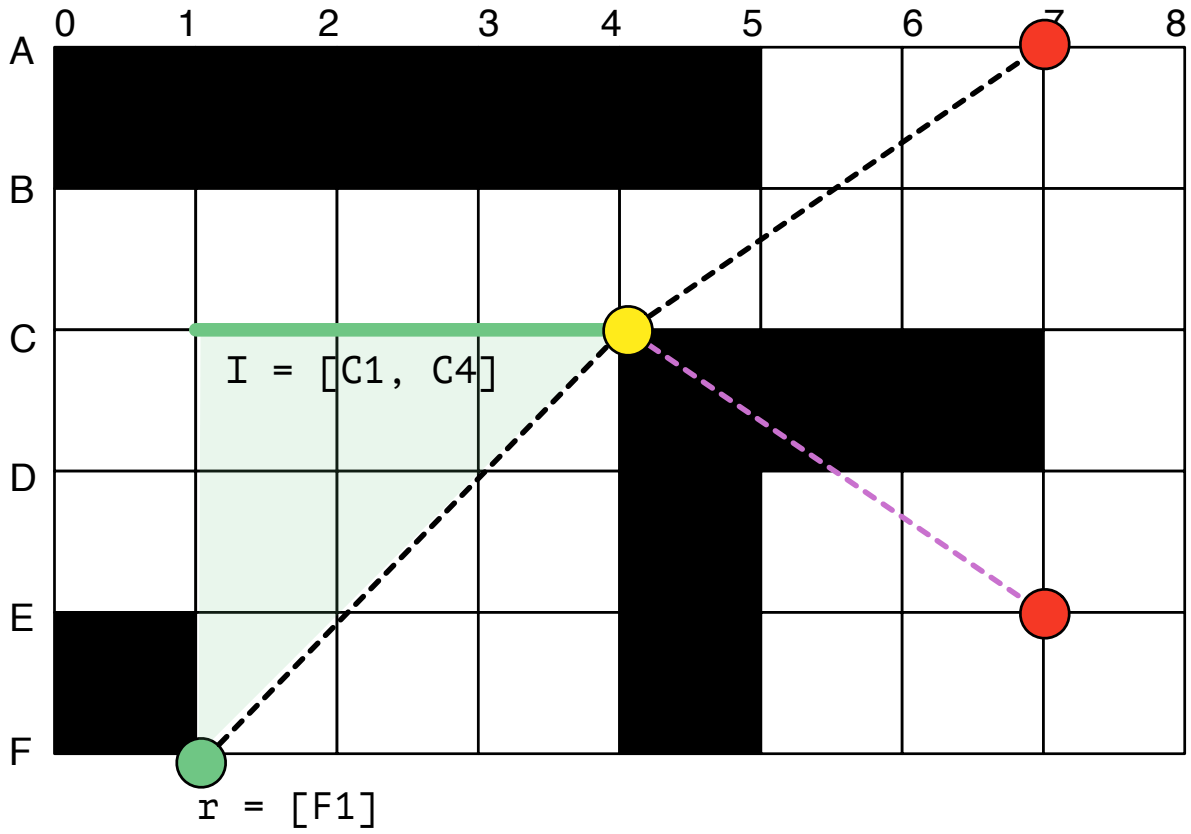


Theoretical properties

Completeness (Sketch)

- Every point is a corner or belongs to an interval.
- Every interval is visible from some predecessor.

Optimality (Sketch)

- Each representative point has a minimum f-value.
- The f-value of each successor is monotonically increasing.
- A node whose interval contains the target is eventually expanded.

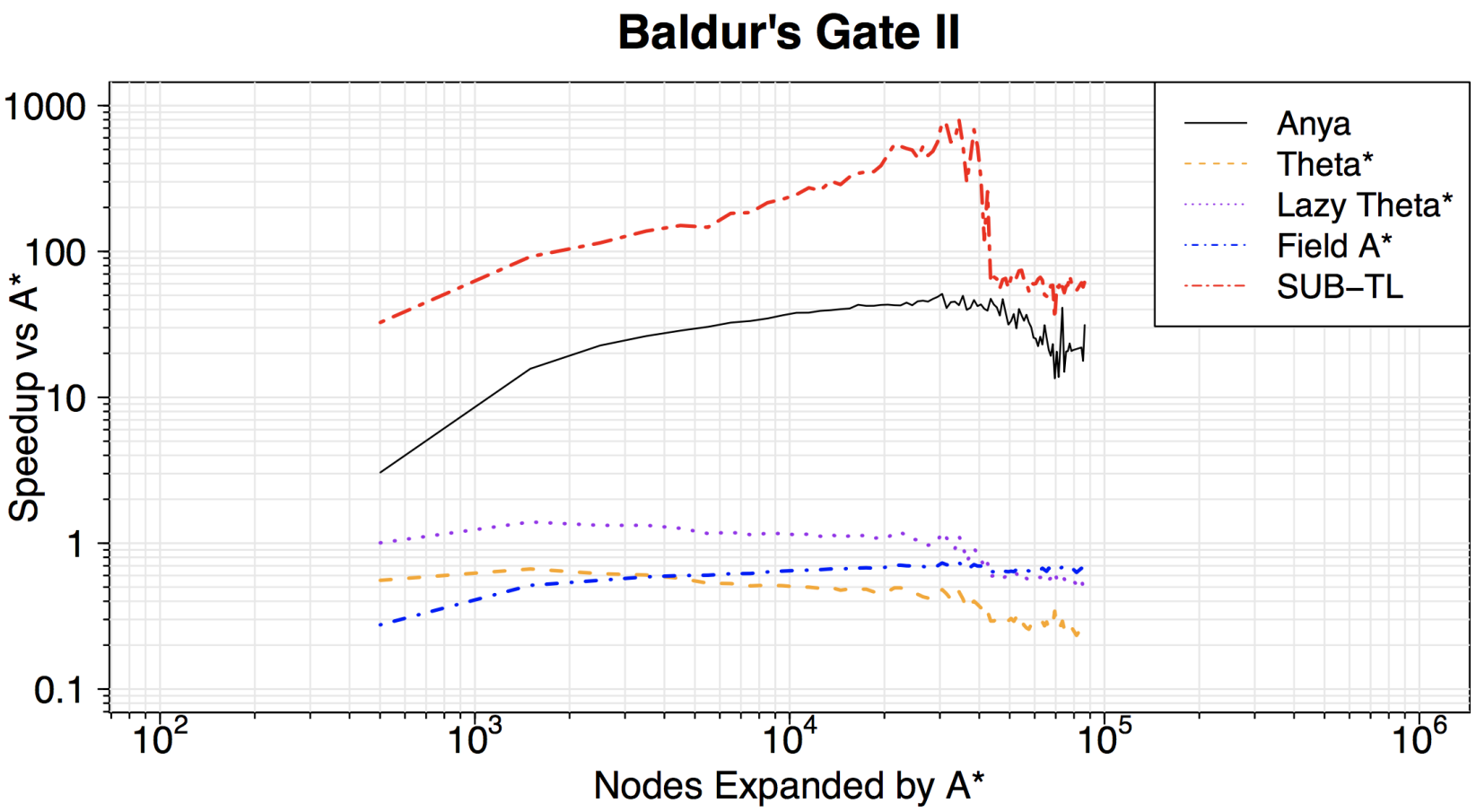
Online

Each search is performed entirely online and without reference to any pre-computed data structures or heuristics.

Full technical details in [Harabor *et al.*, 2016].

Results on Games Maps

Speedup (time) vs grid A* on a range of game benchmarks appearing in Nathan Sturtevant's repository at `http:://movingai.com`.



75 maps, 93,160 problem instances.

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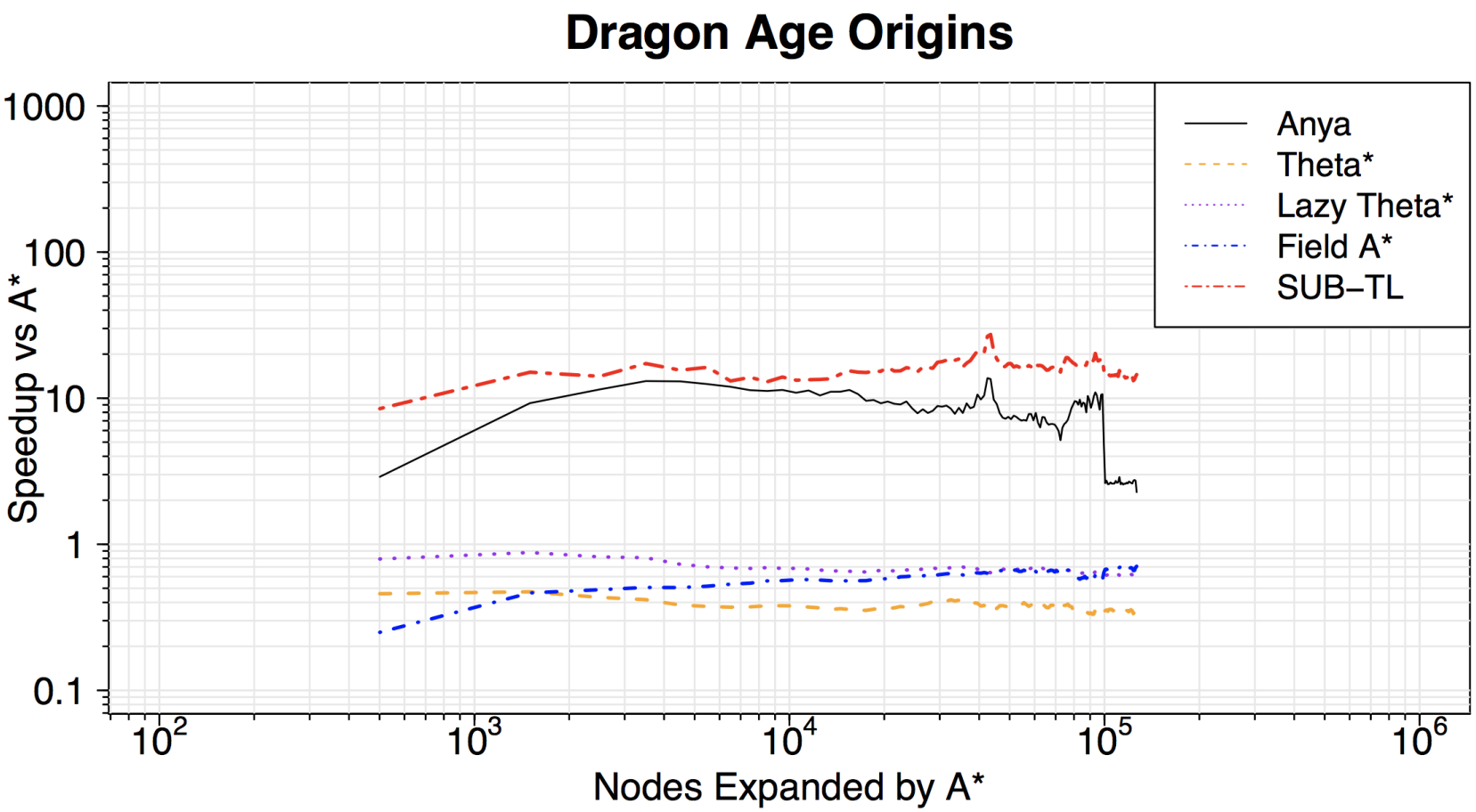

156 maps, 159,465 problem instances.

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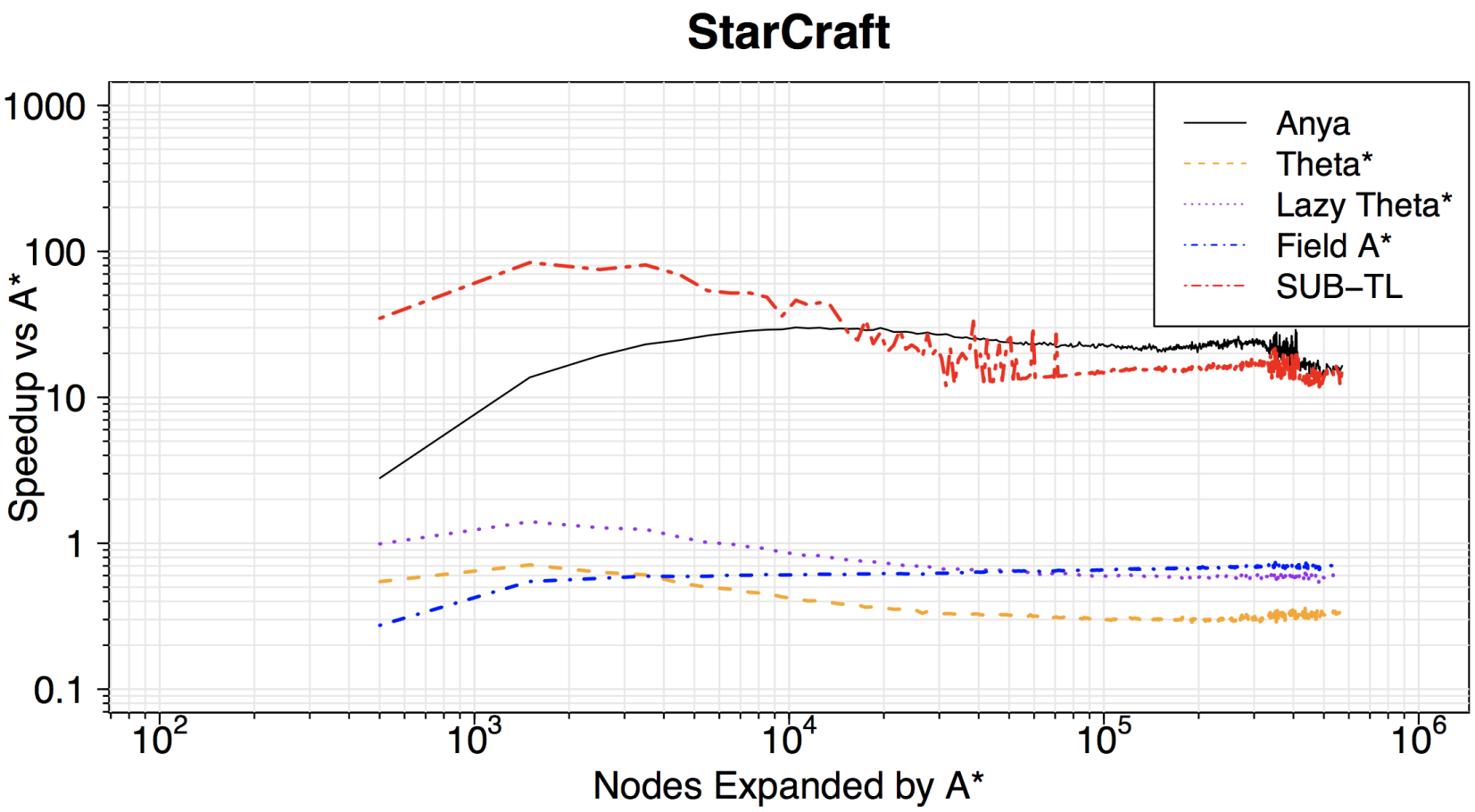


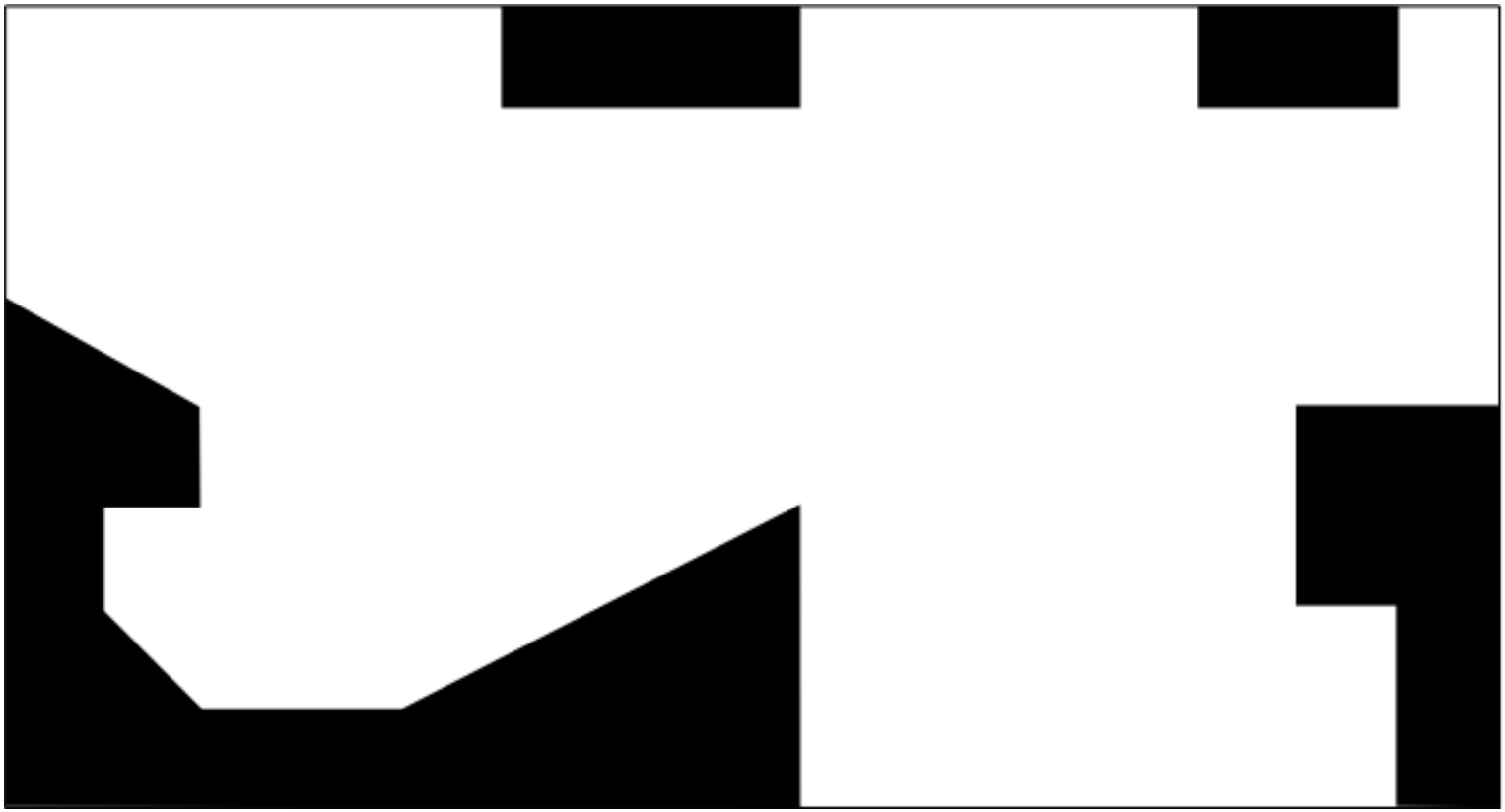
75 maps, 198,230 problem instances.

Euclidean Shortest Paths in 2D

Euclidean Shortest Path Problems in 2D

2D ESPP

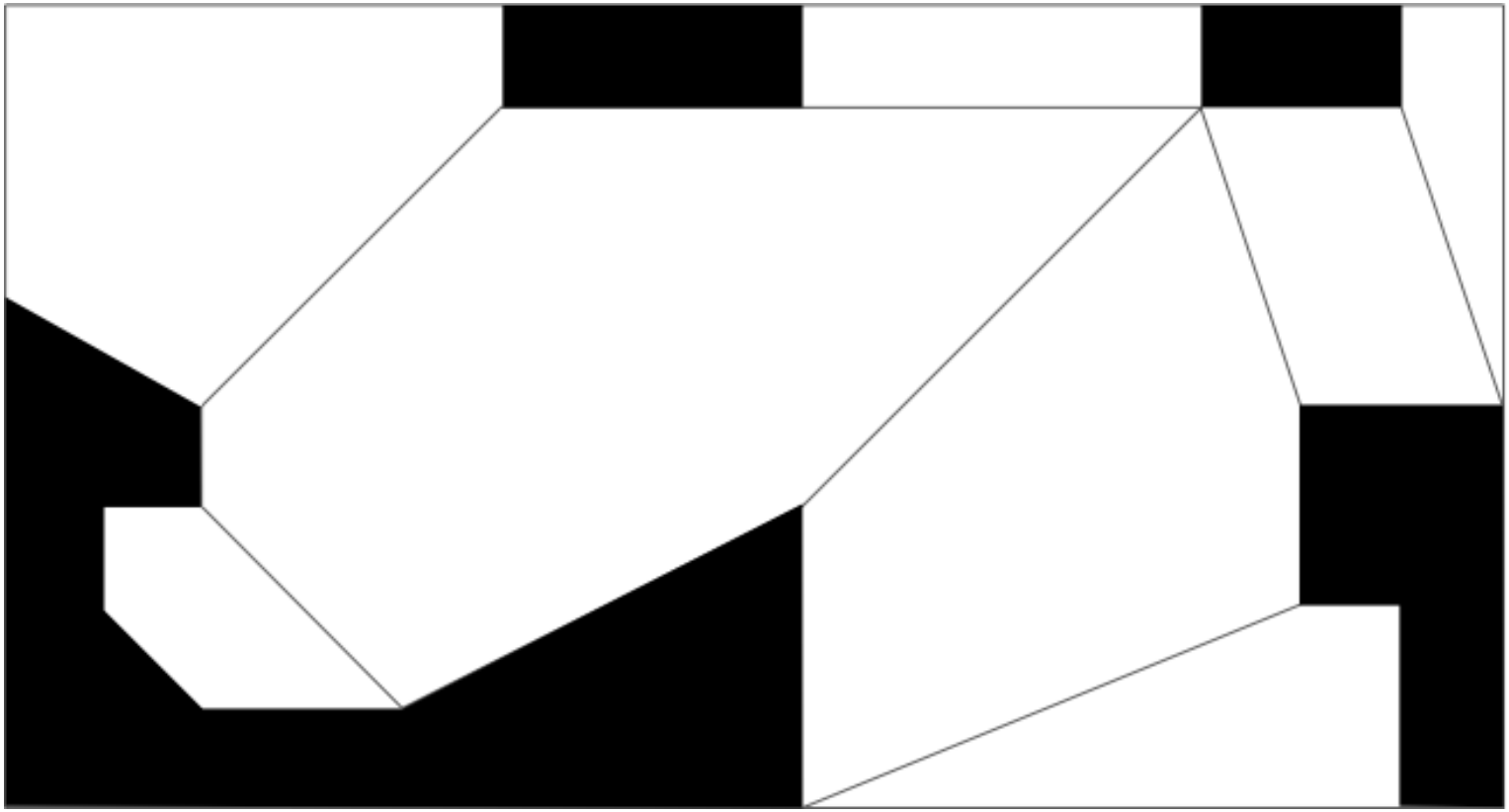
Find a shortest path among a set of polygonal obstacles.



2D ESPP on a Navigation Mesh

Navigation Mesh

A partitioning of the traversable space into a collection of convex polygons. Cheap to build. Common in many application areas.



From Any-angle Pathfinding to ESPP

Polyanya

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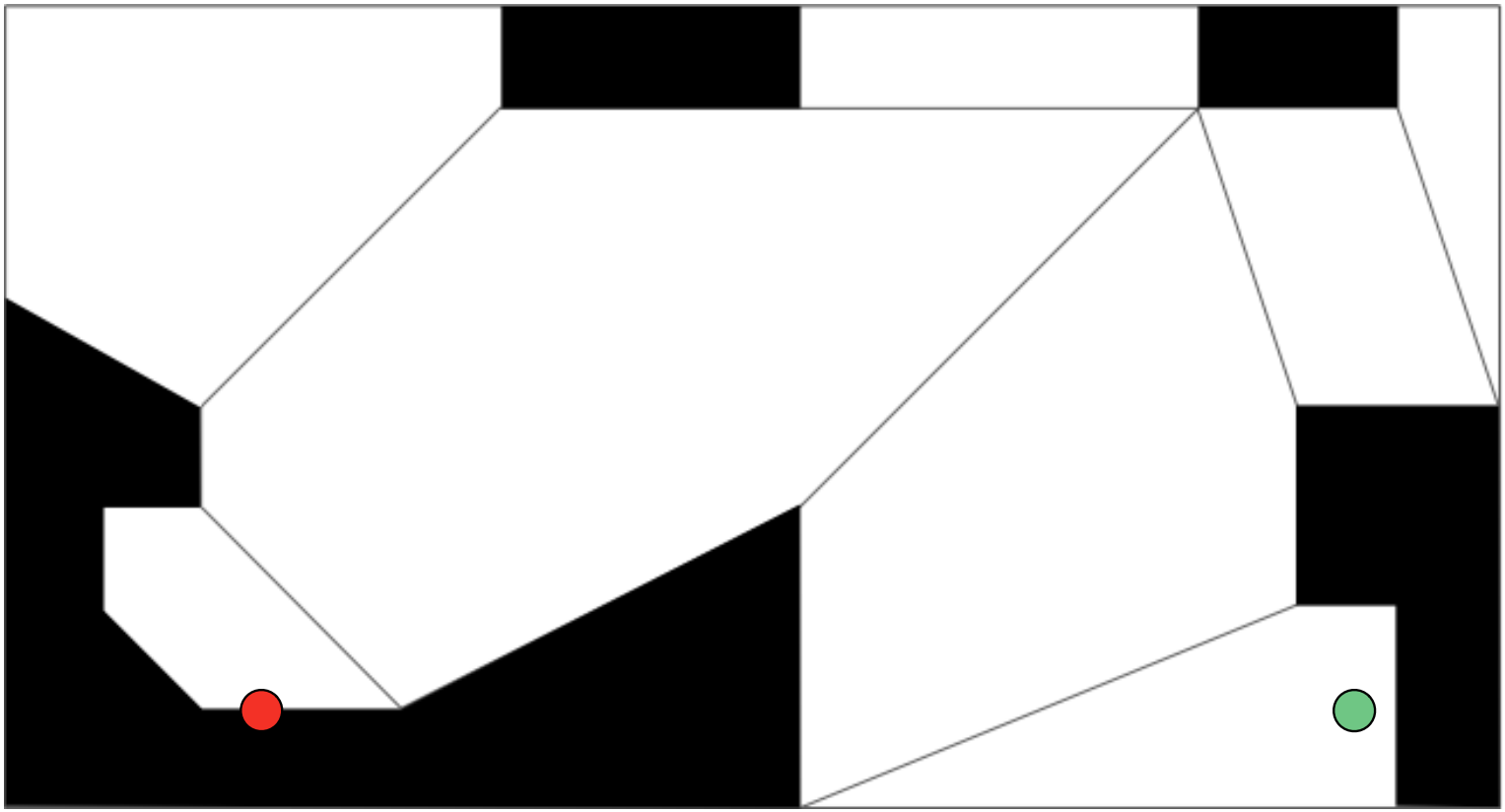

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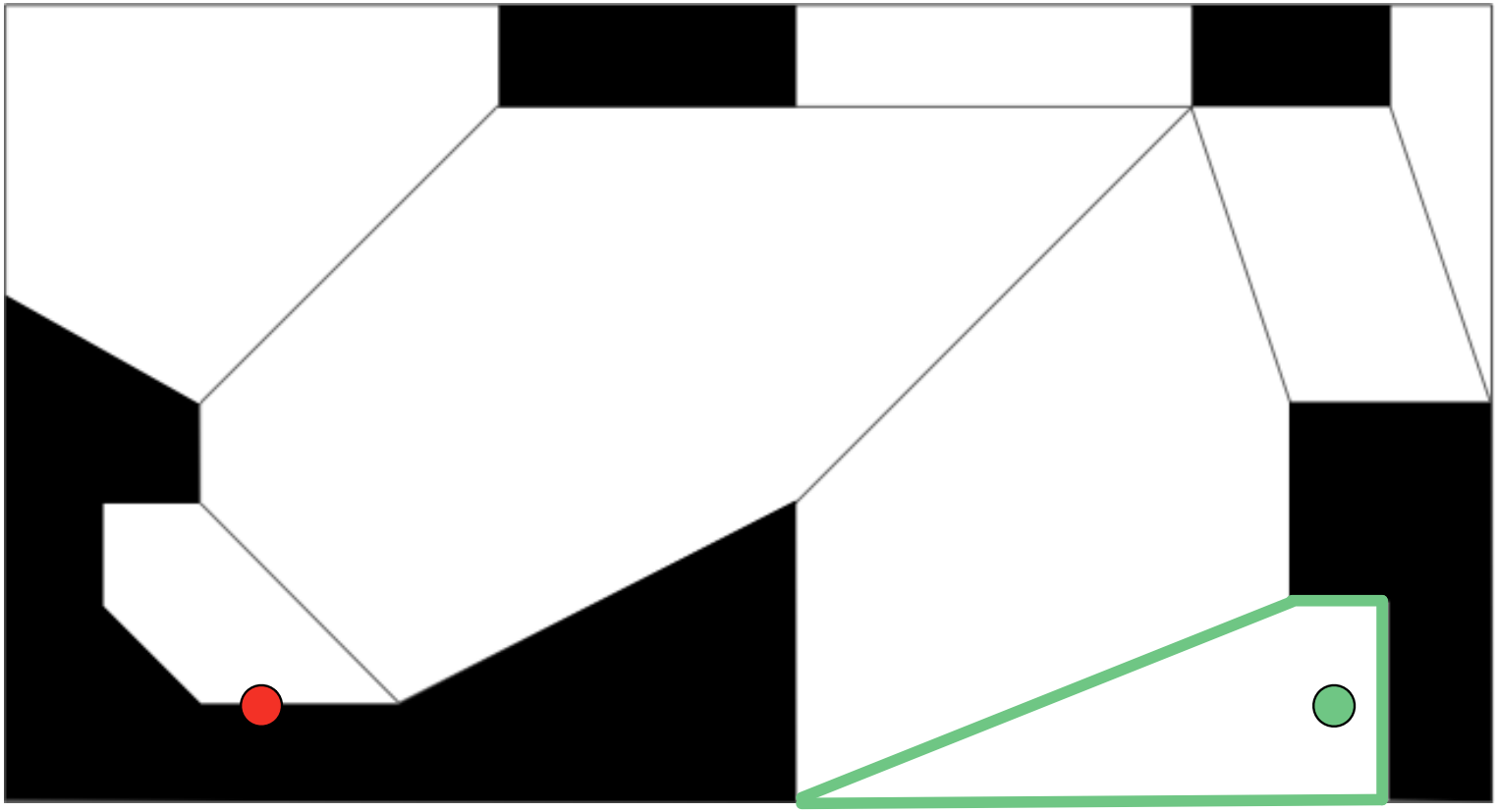

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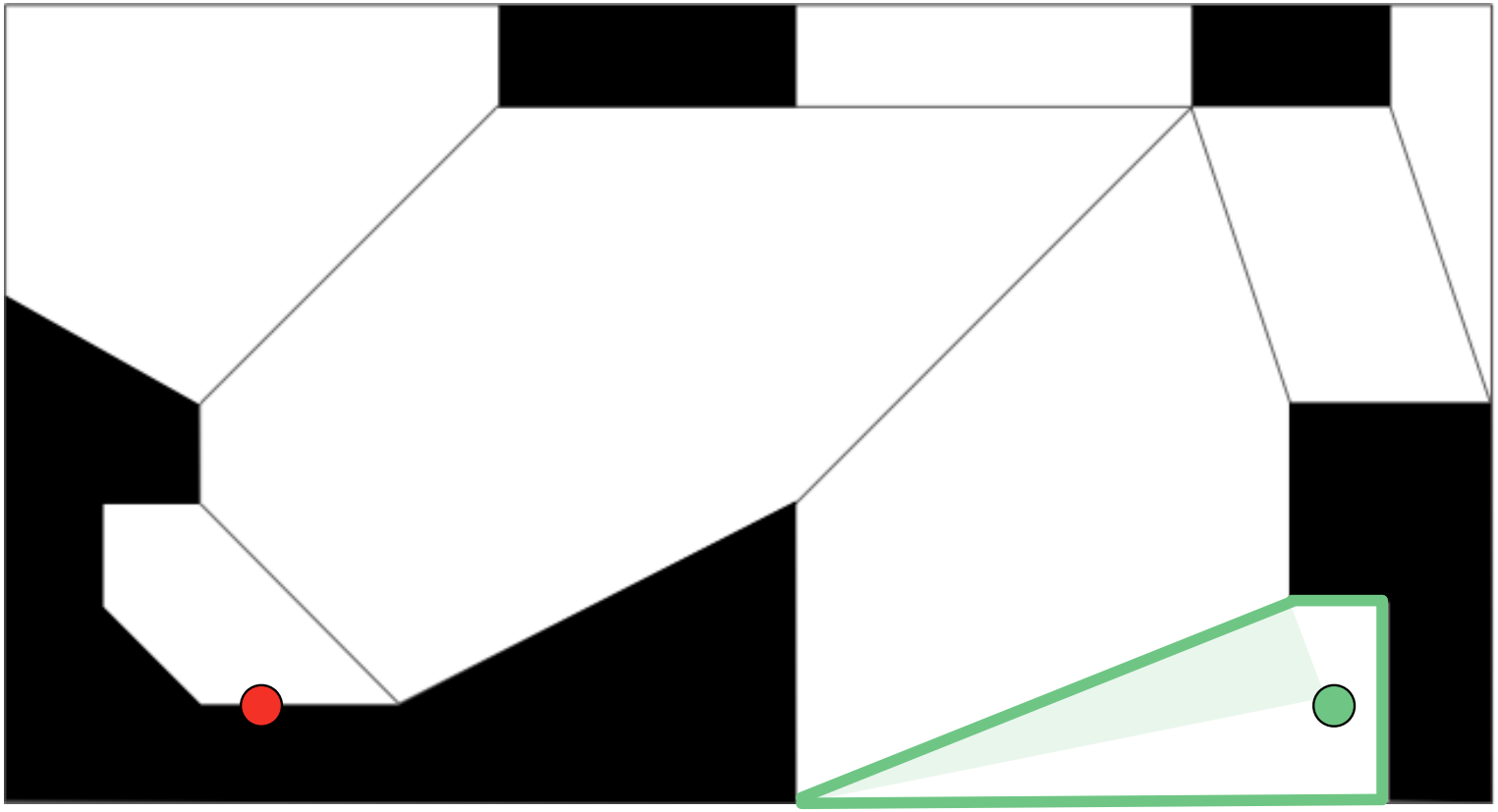

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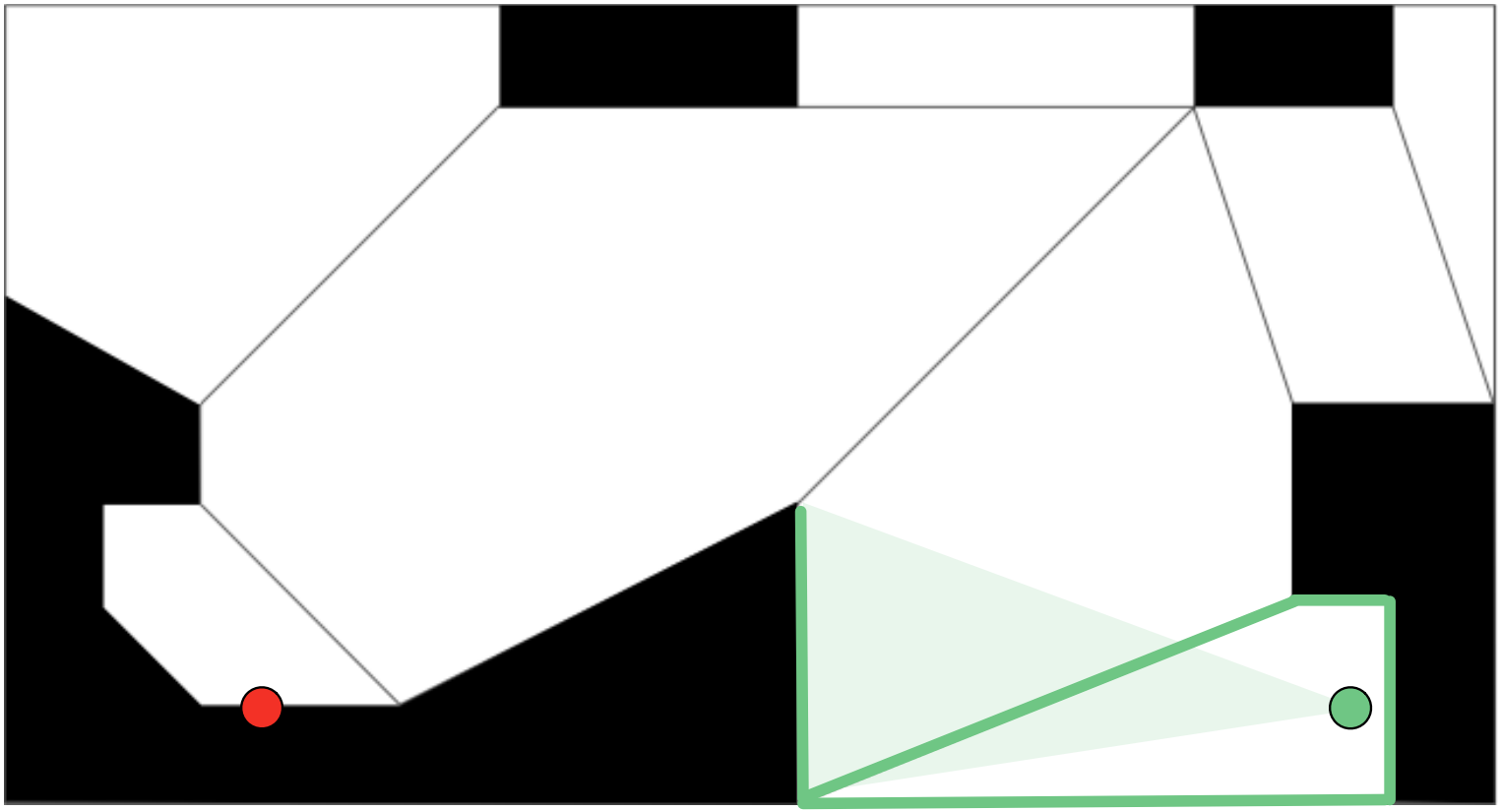

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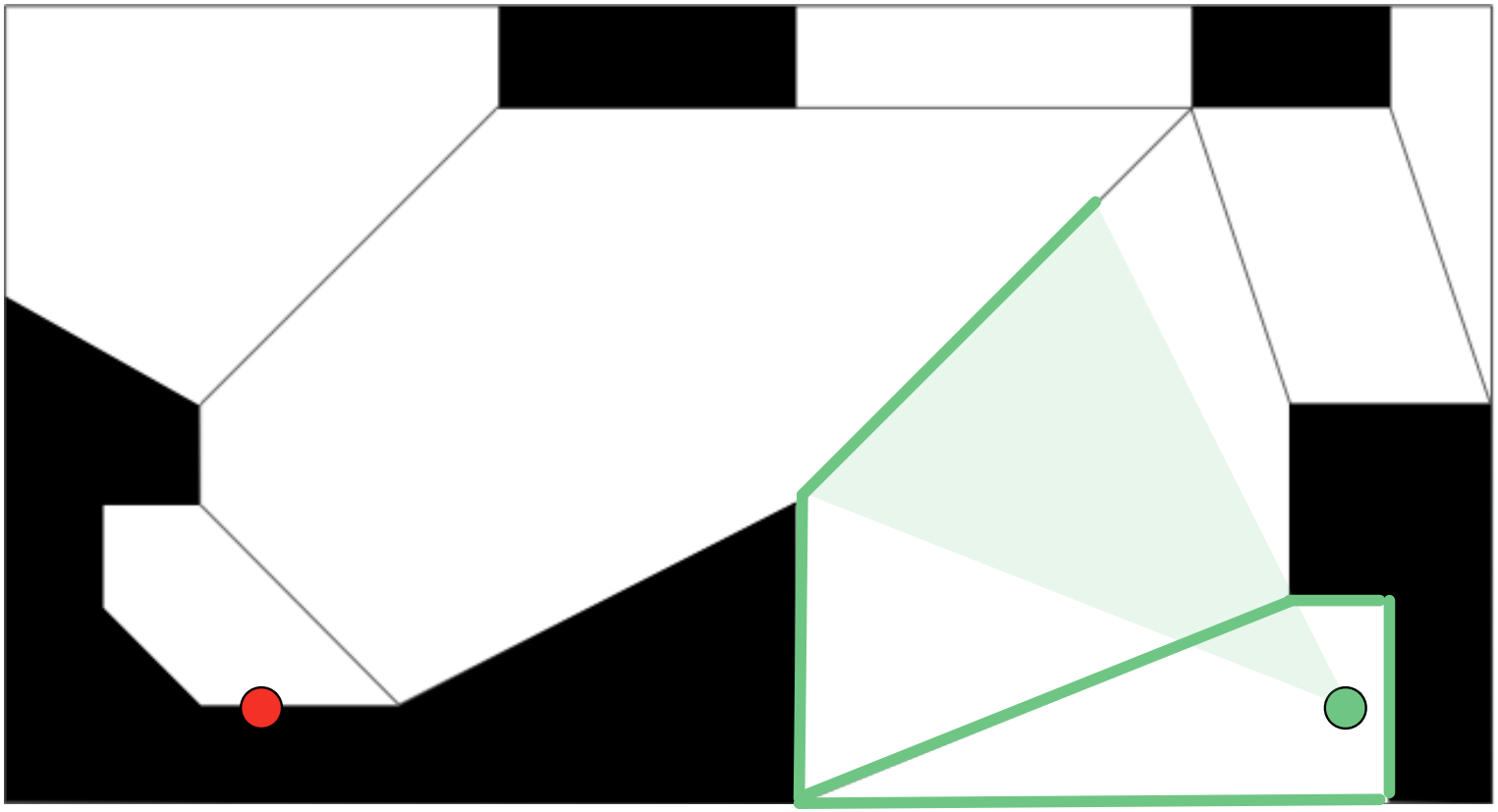

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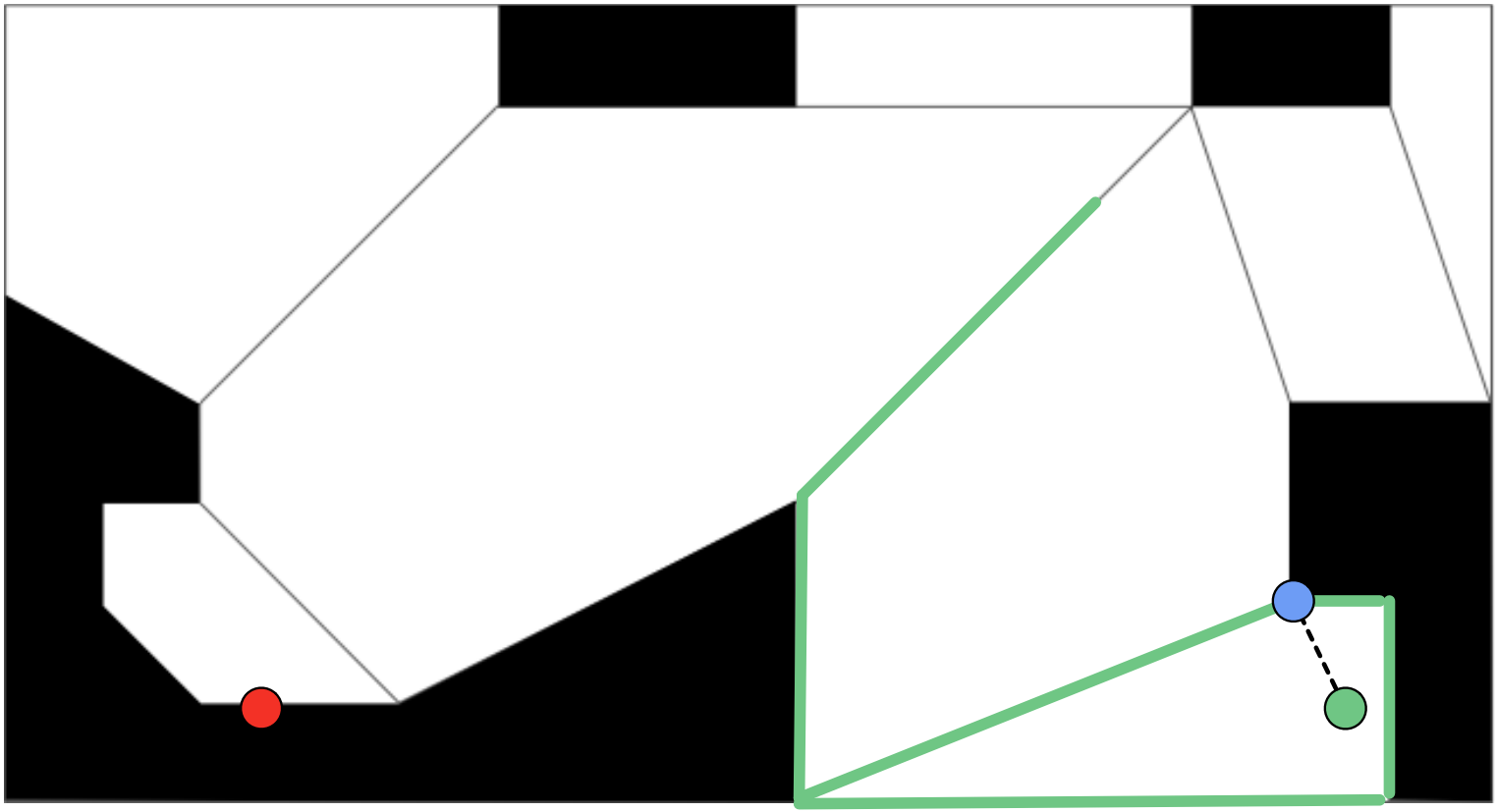

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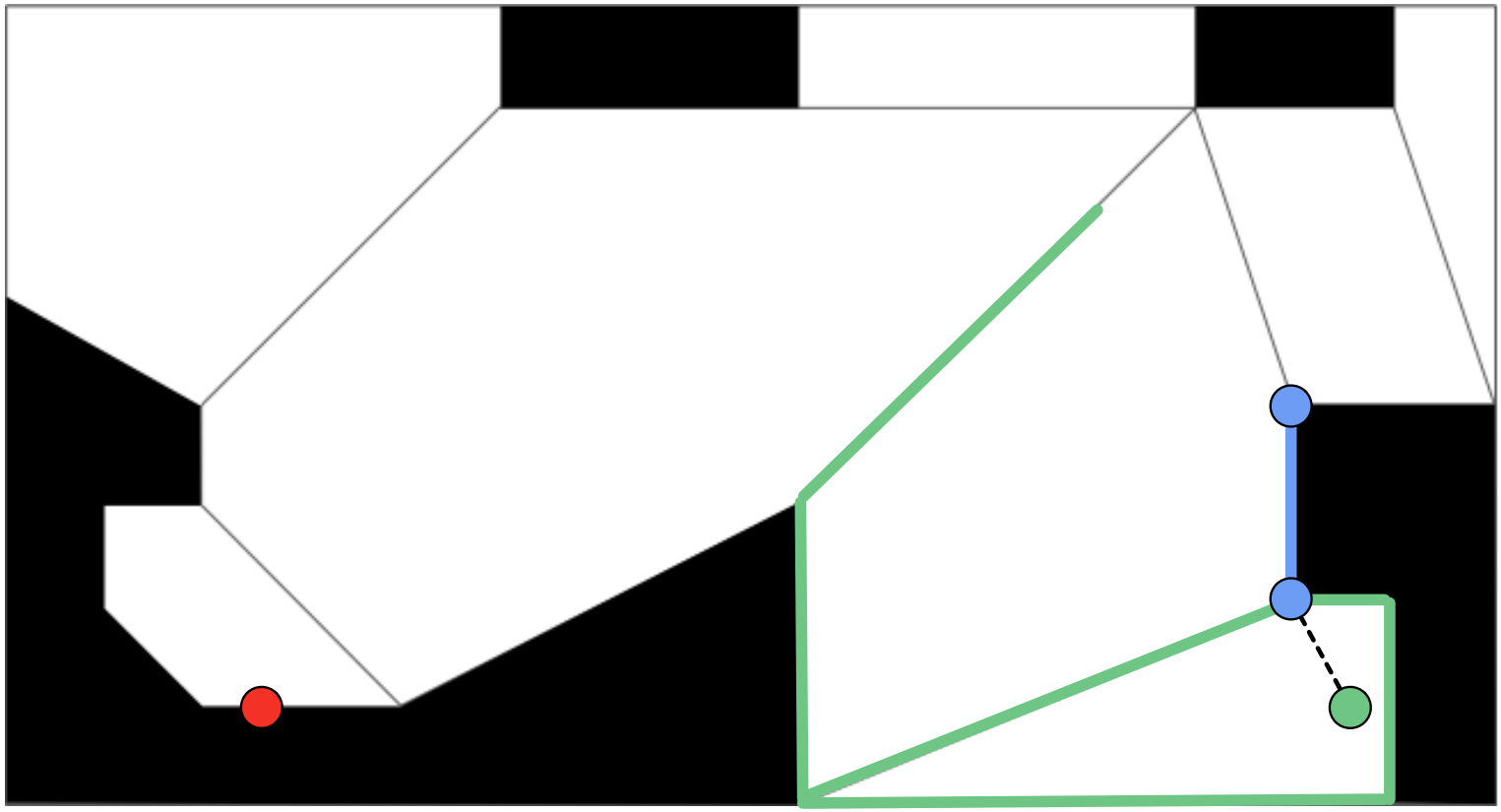

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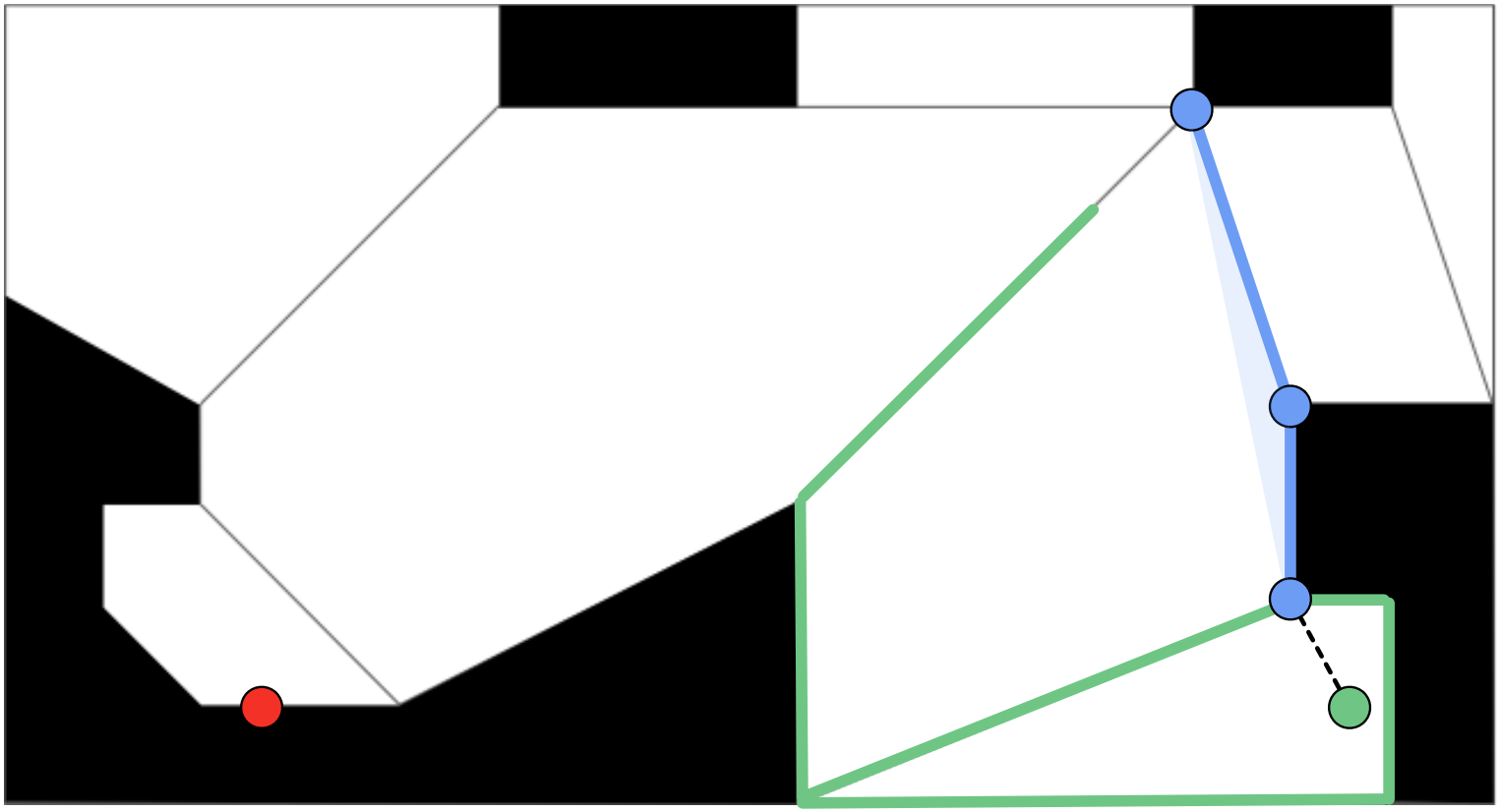

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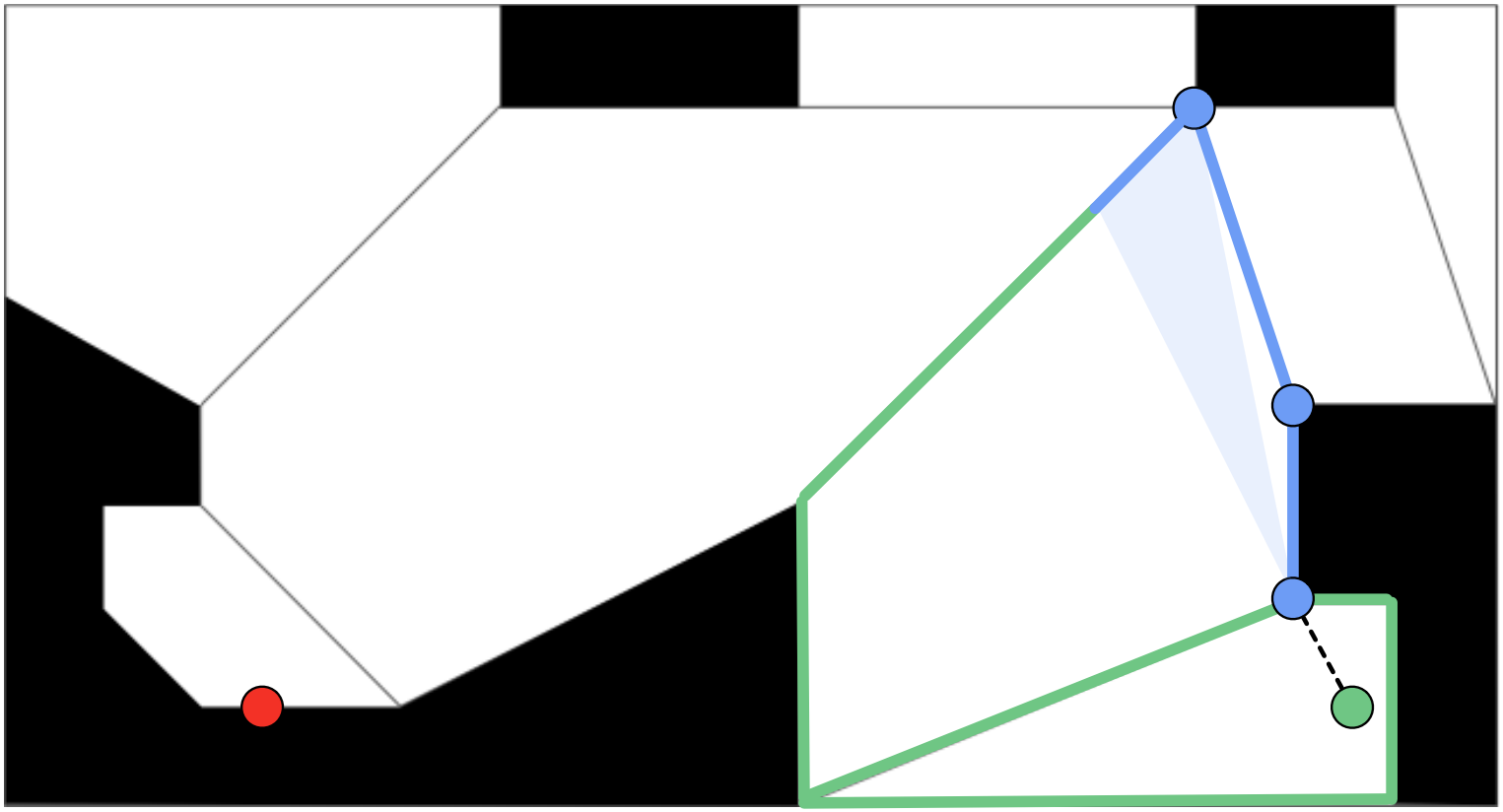

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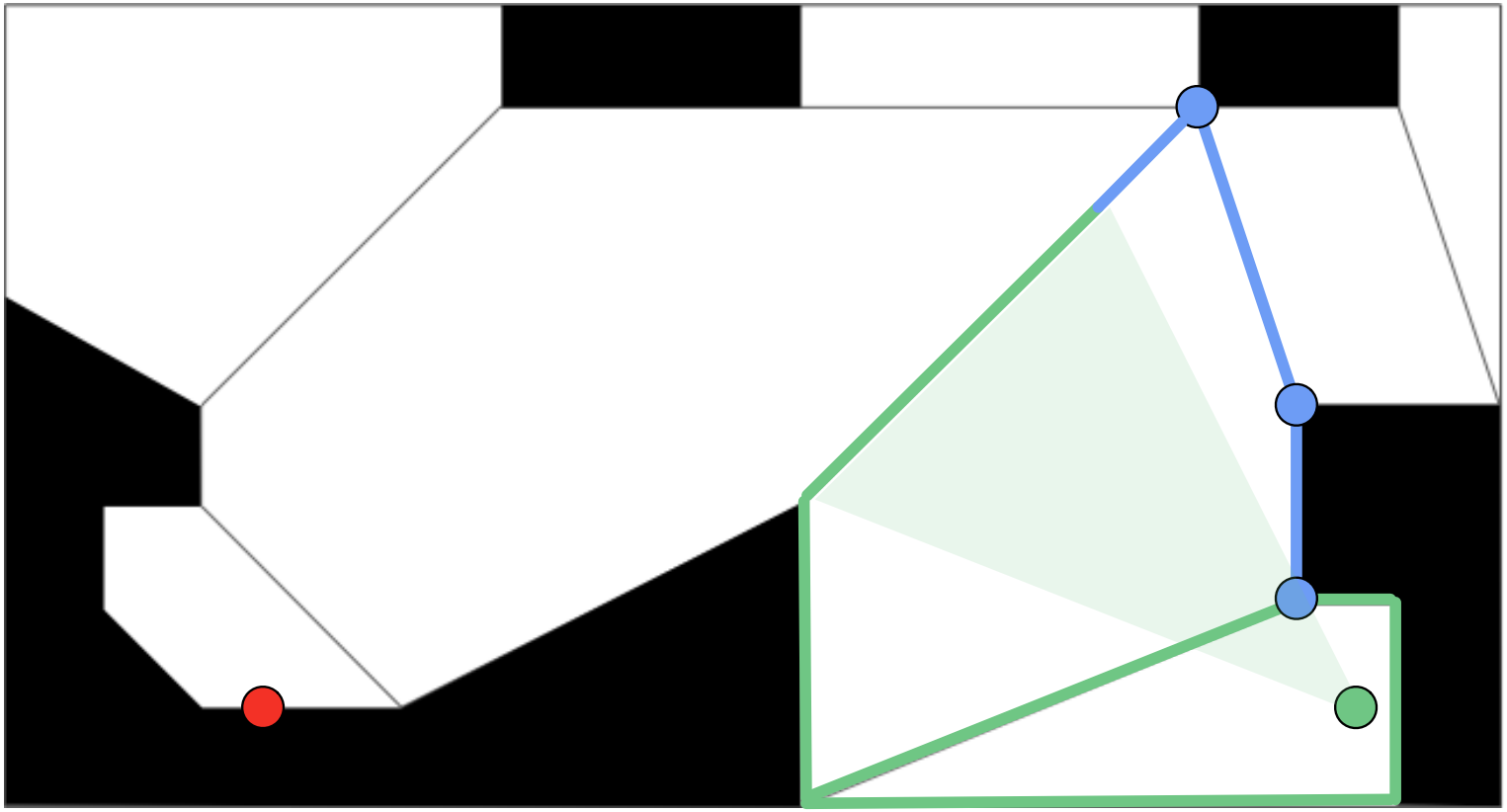

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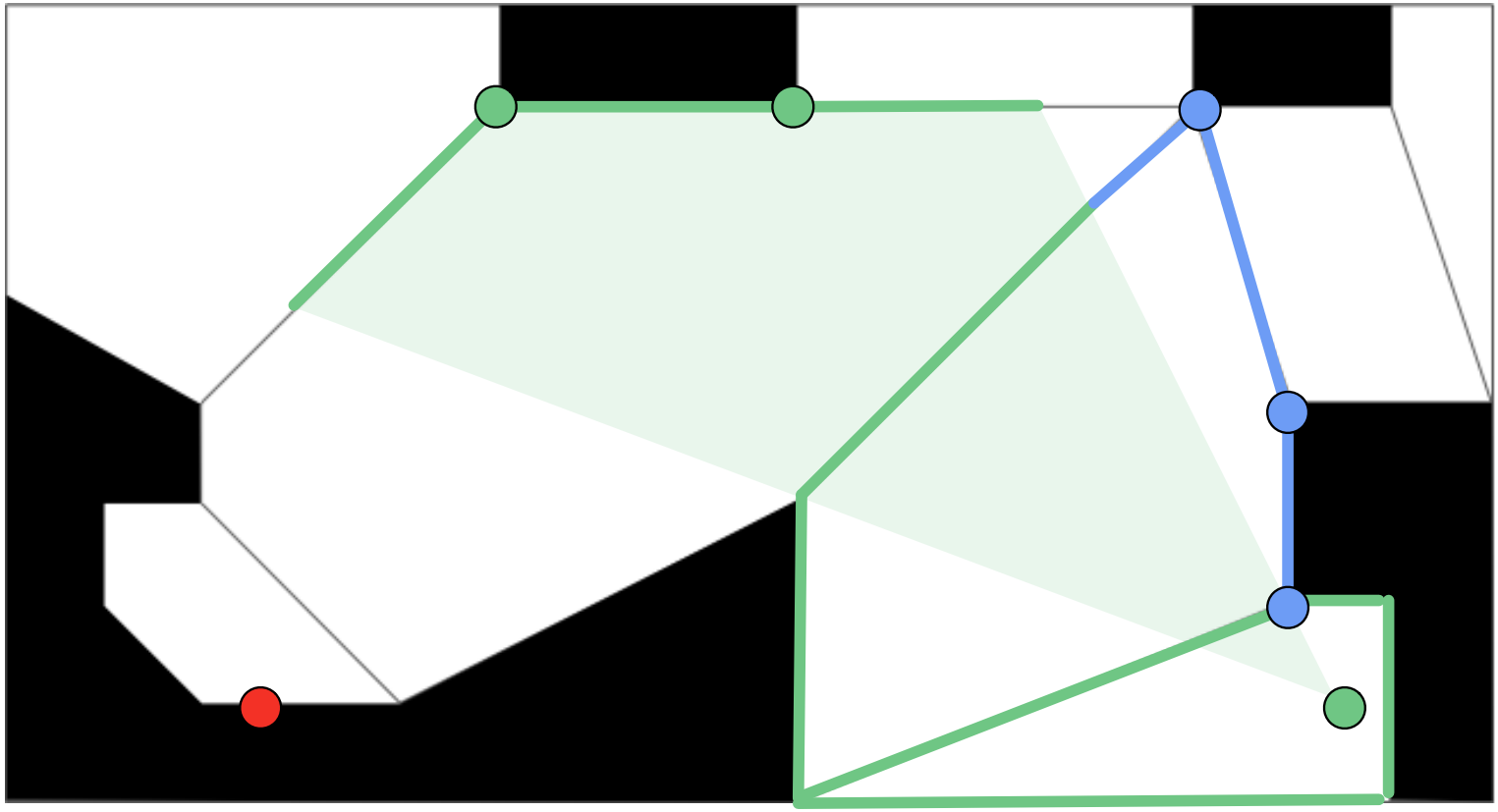

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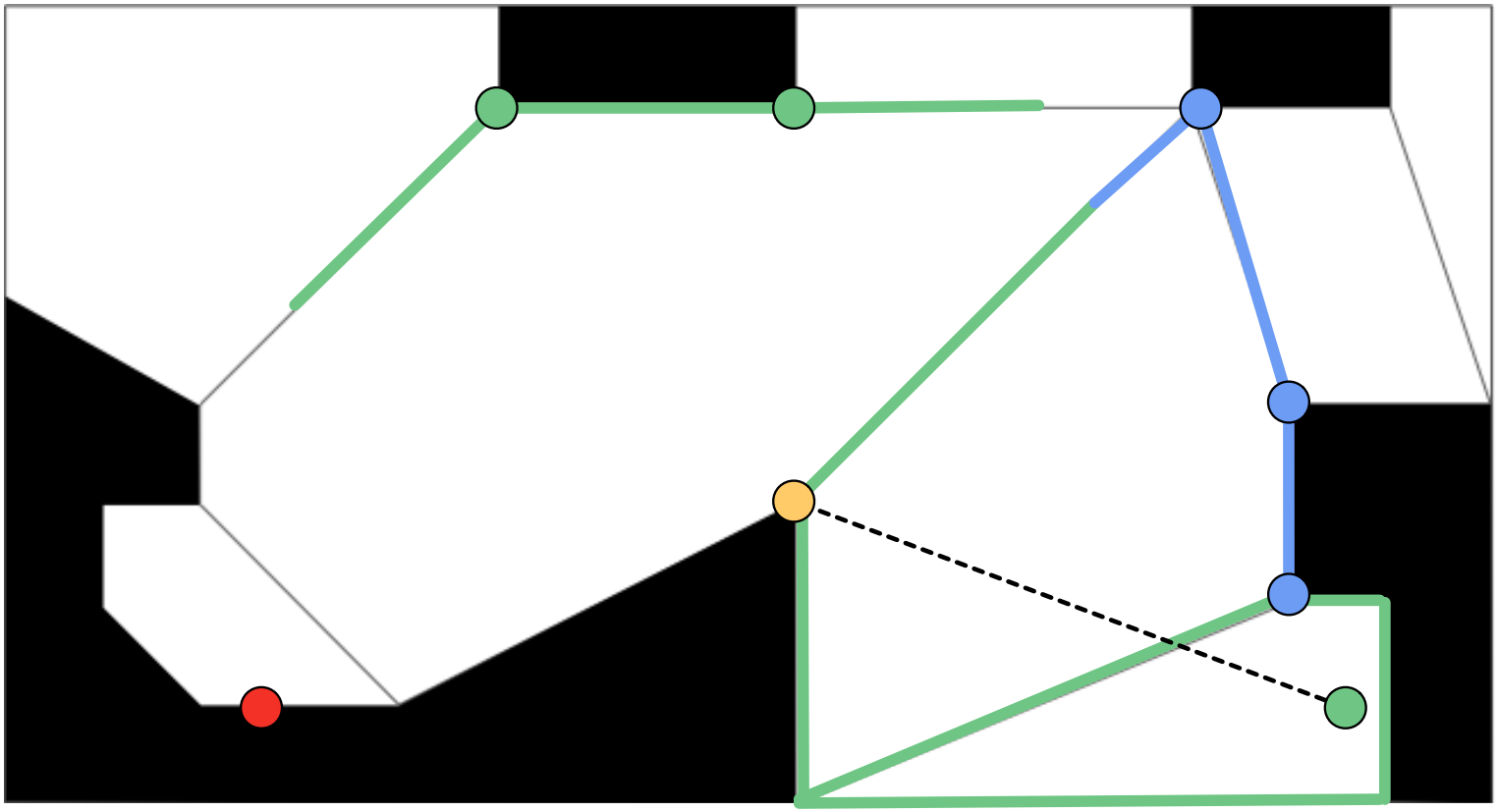

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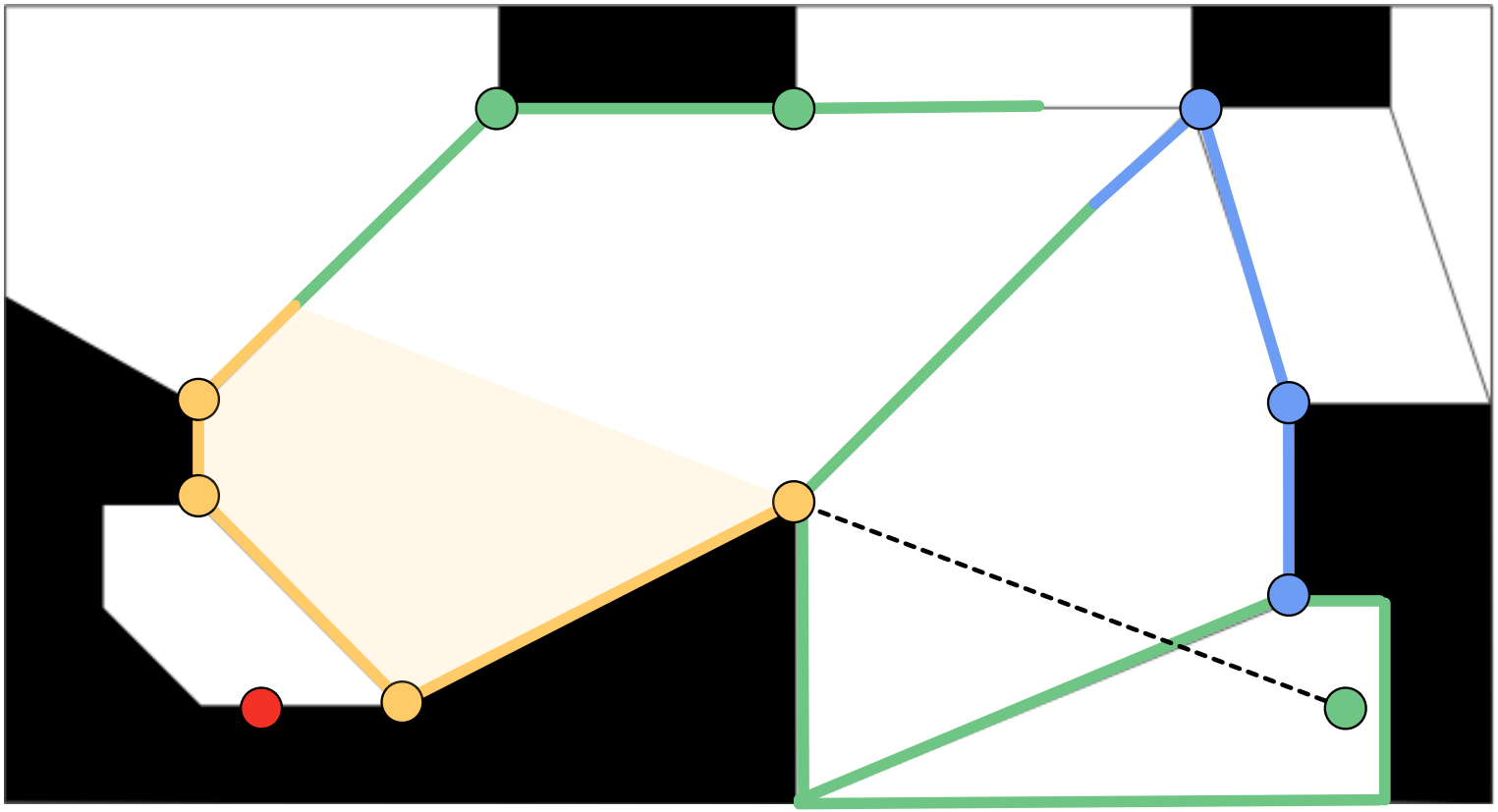

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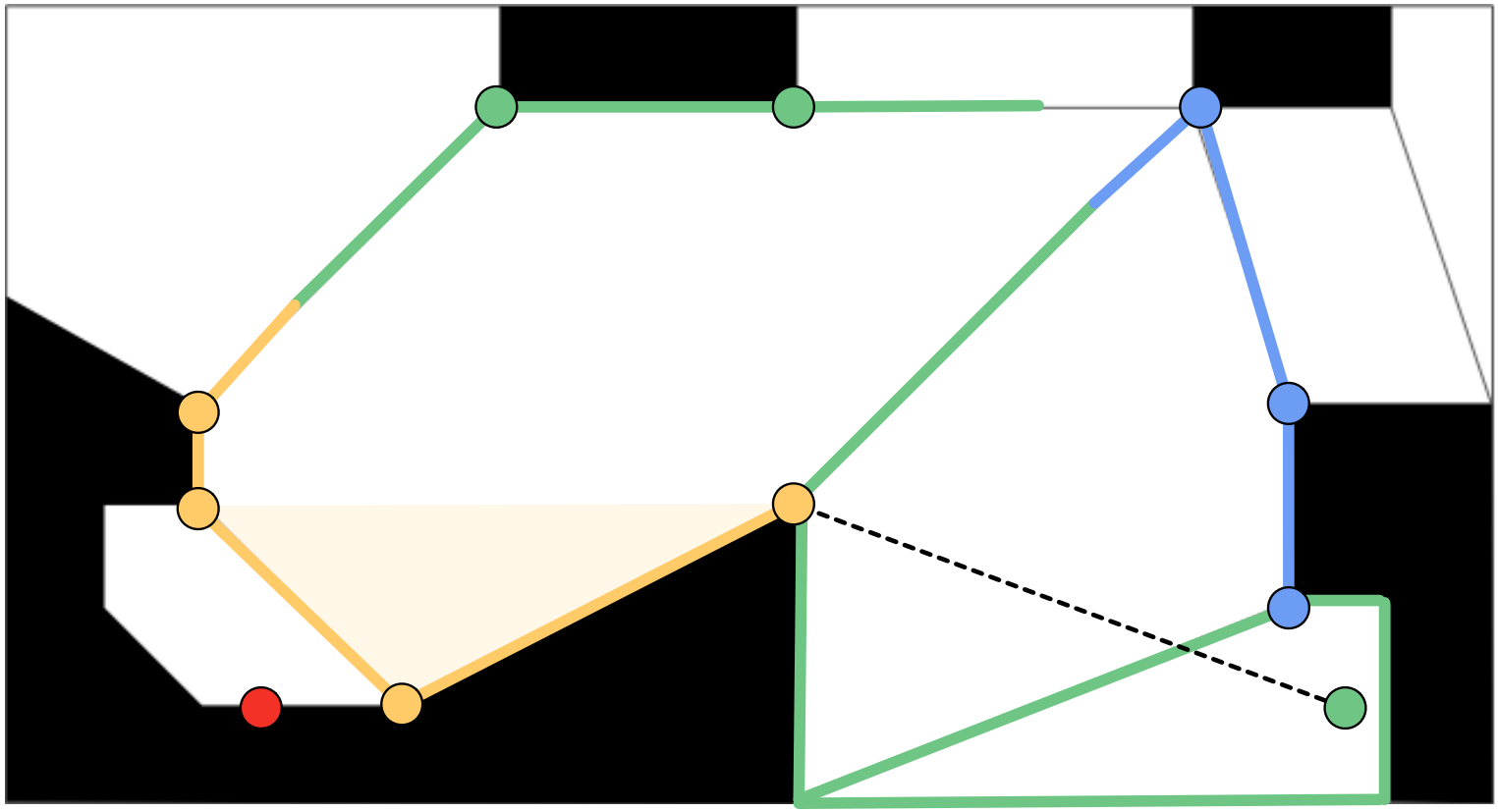

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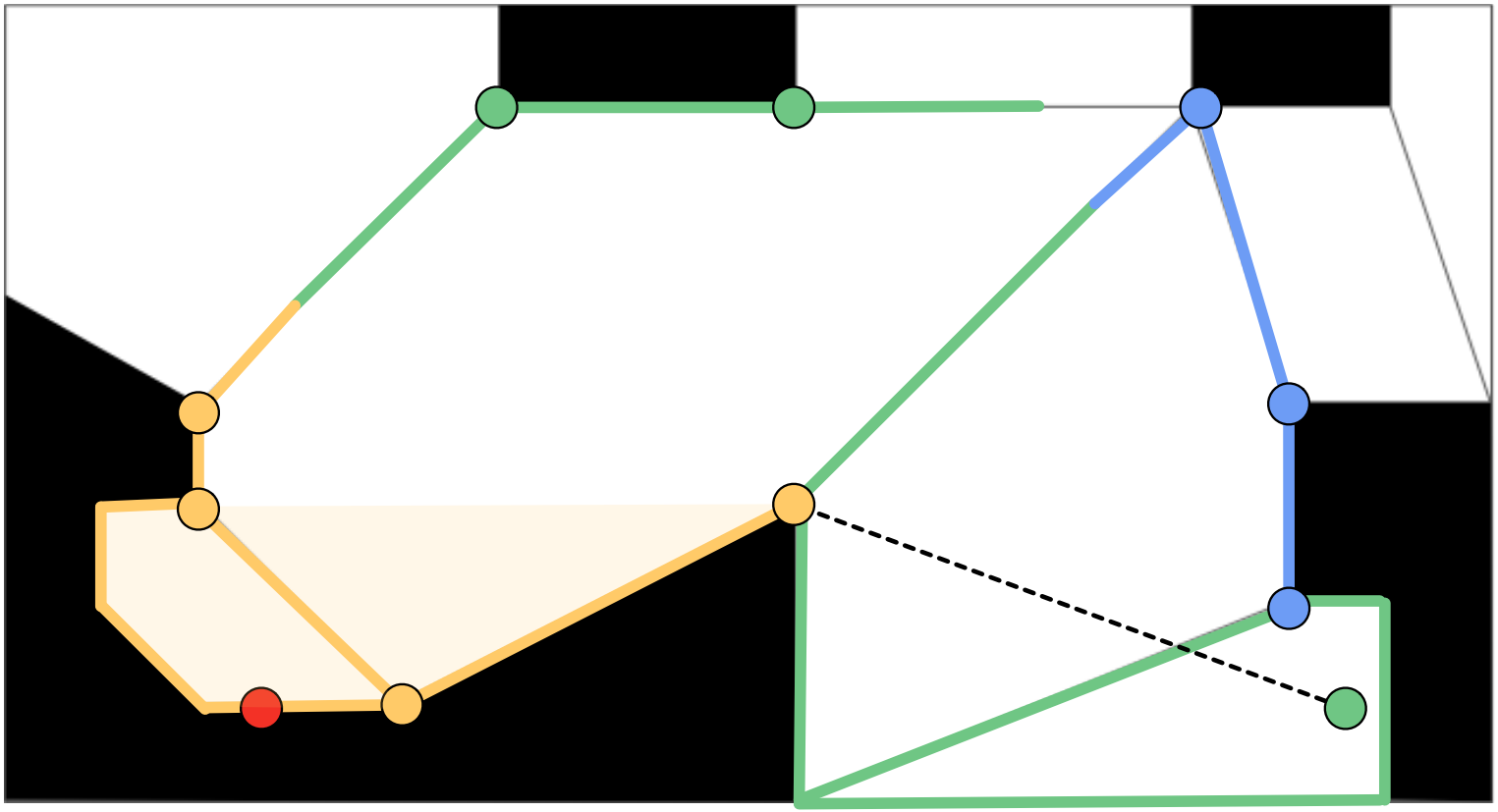

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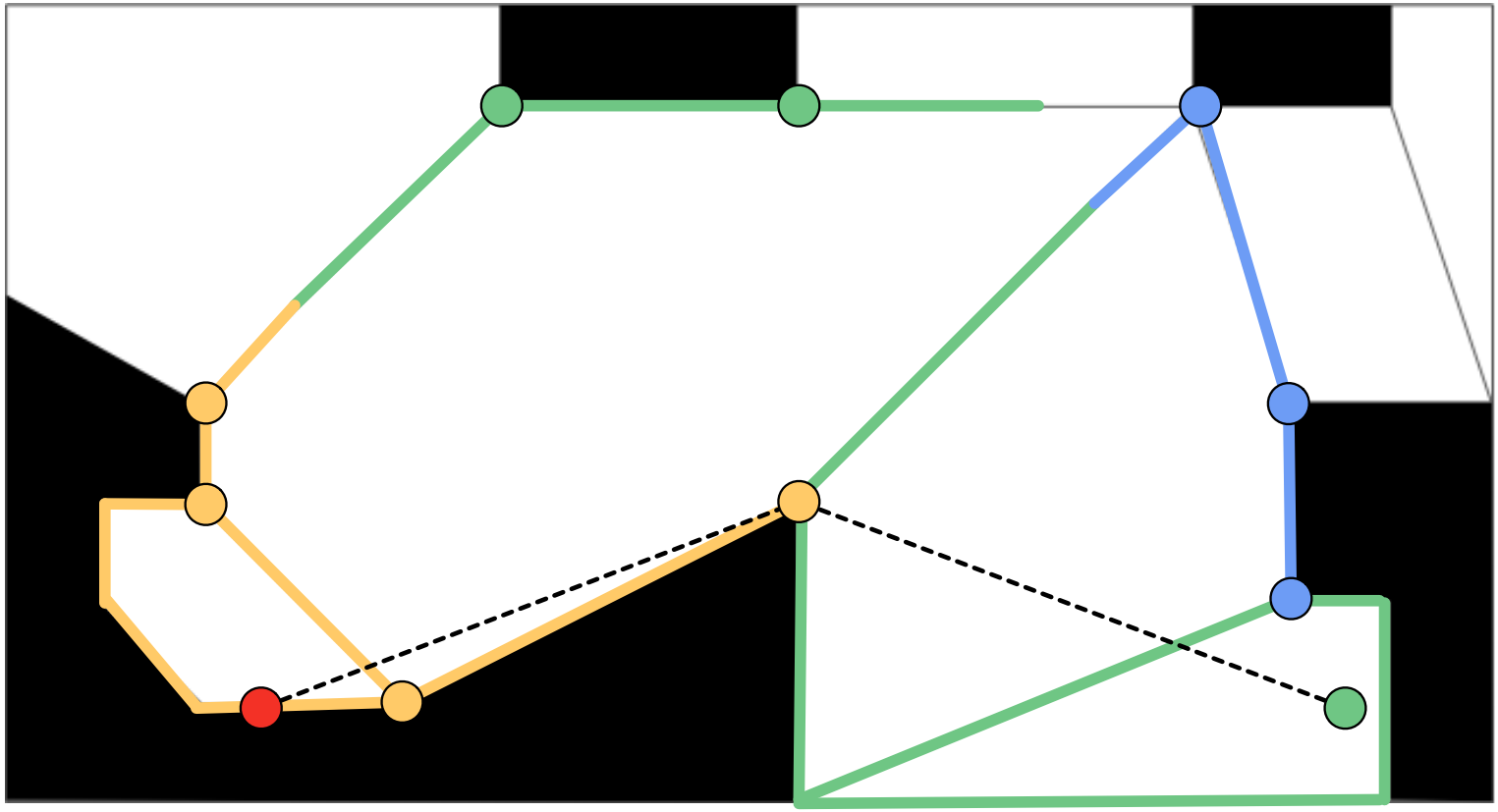
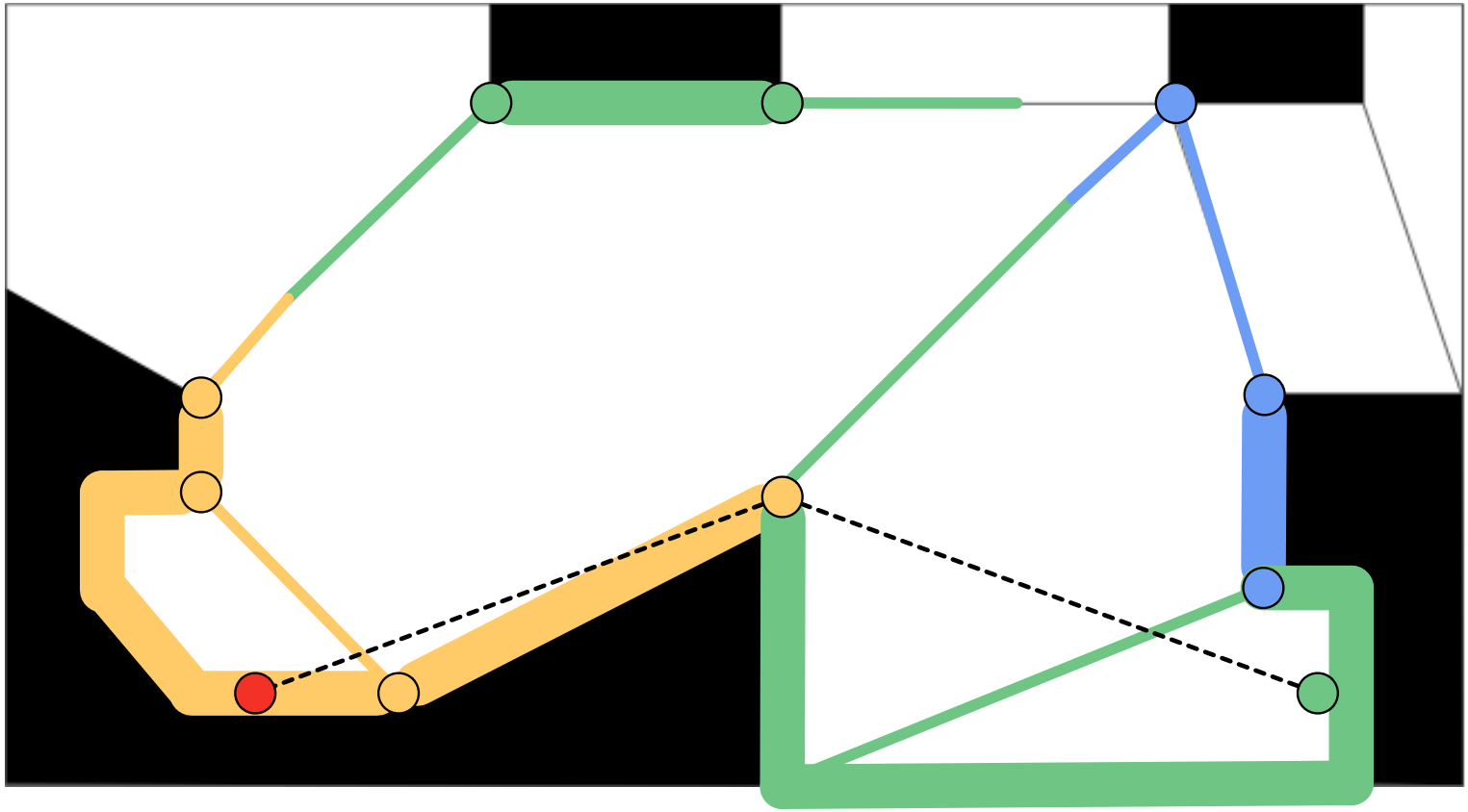


Further optimisations

Dead-end pruning

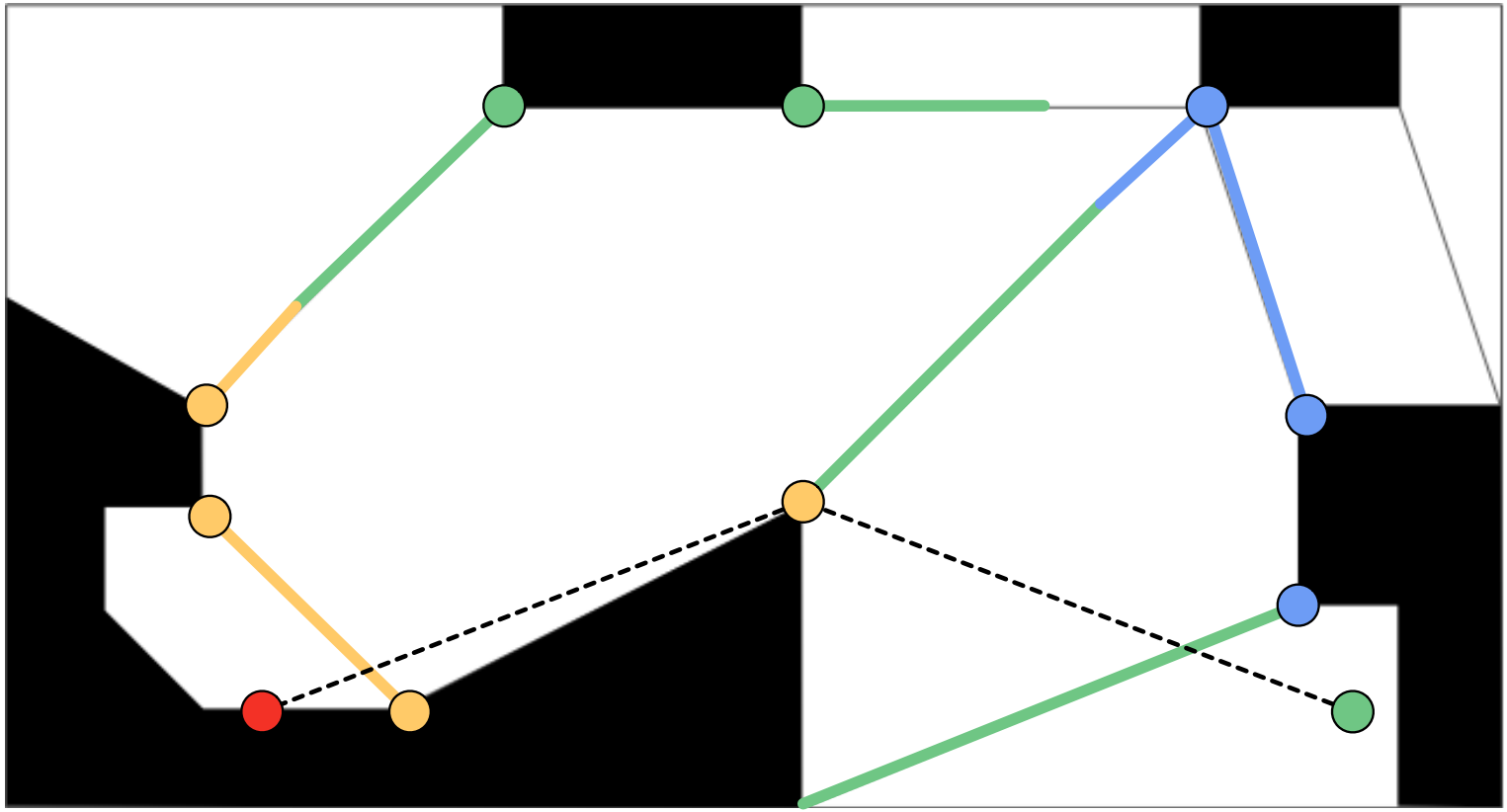
Prune all nodes that "push" into obstacles or into polygons that have only one entry edge.



Further optimisations

Dead-end pruning

Prune all nodes that "push" into obstacles or into polygons that have only one entry edge.



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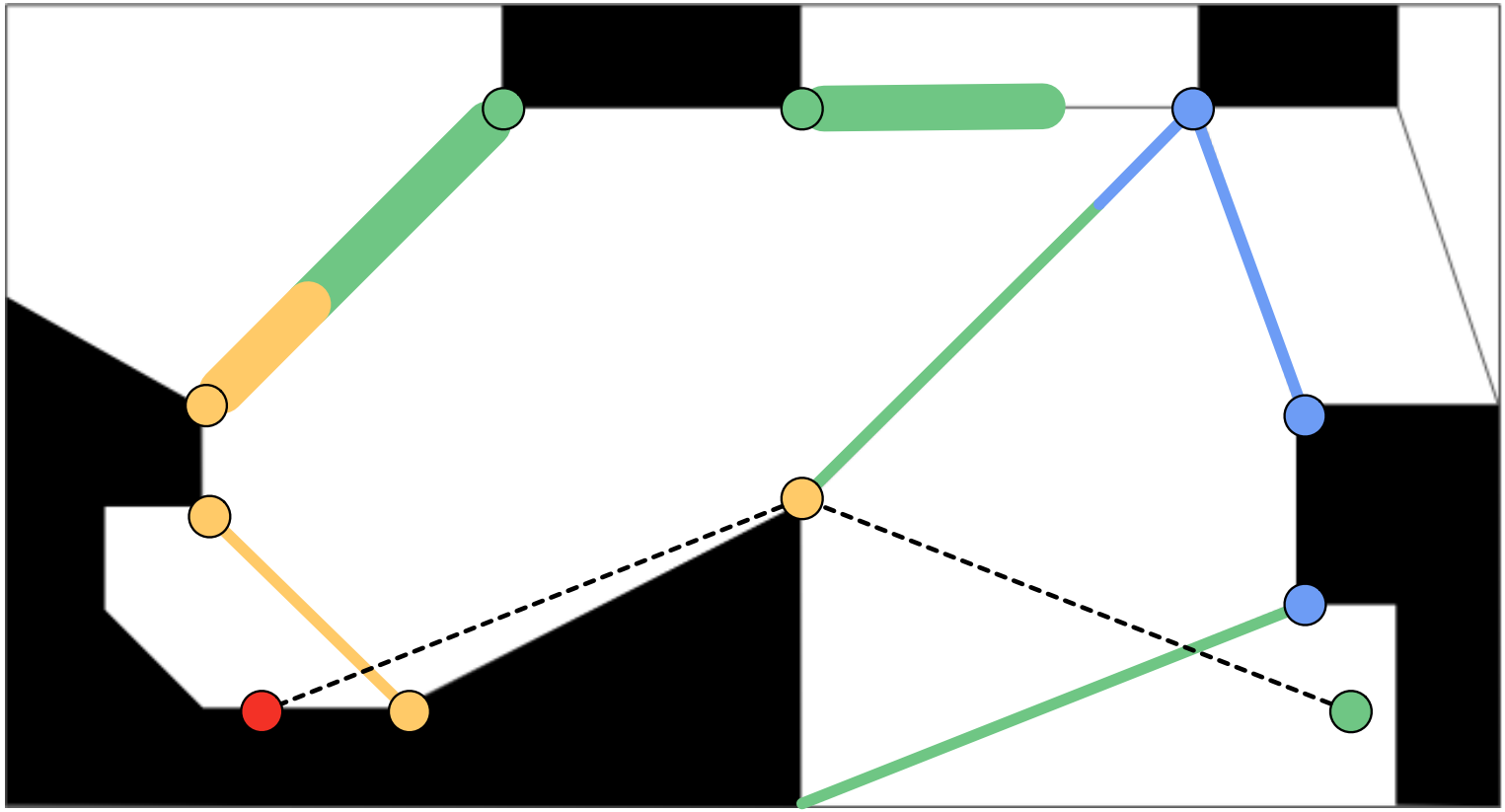

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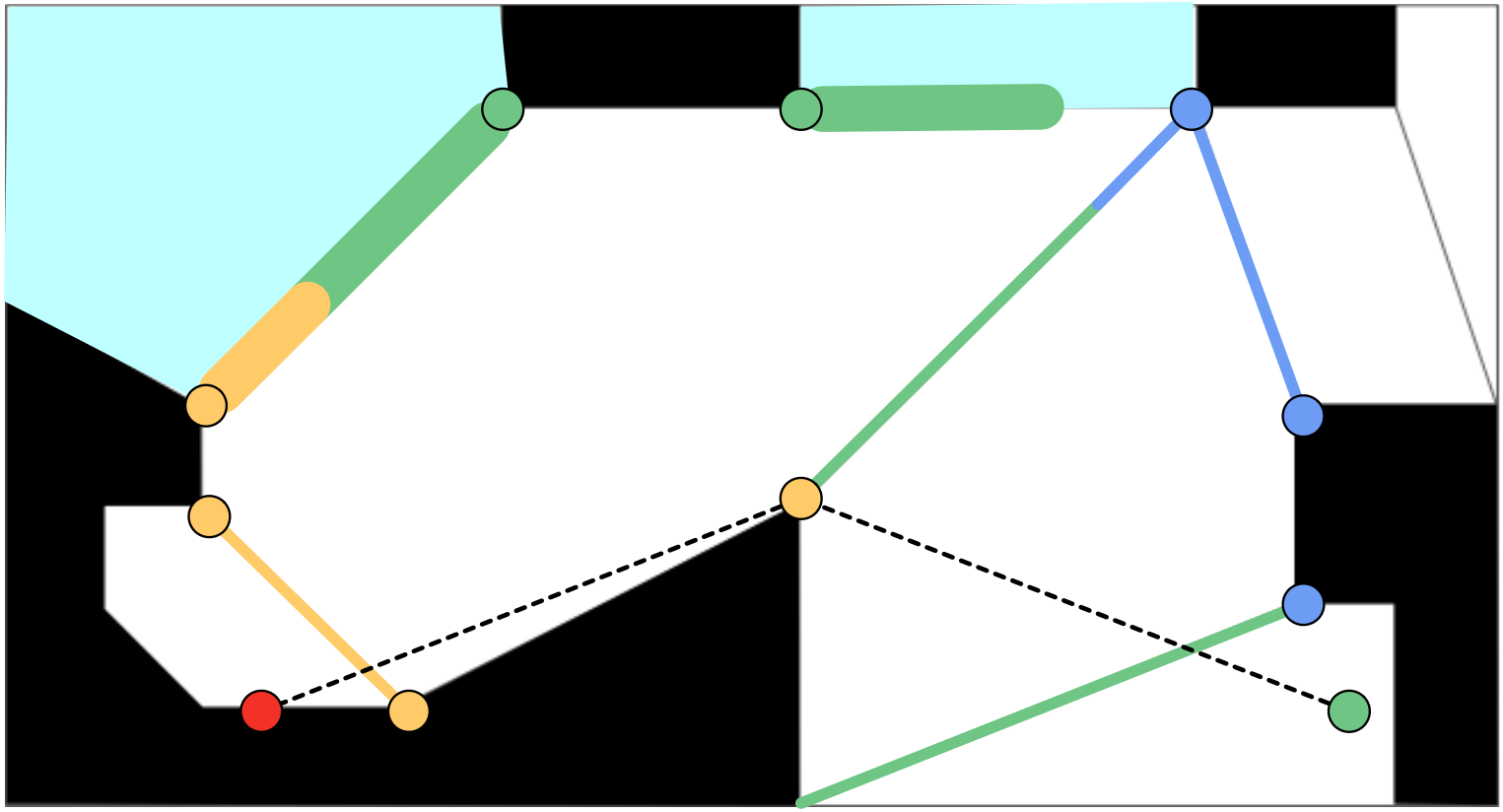

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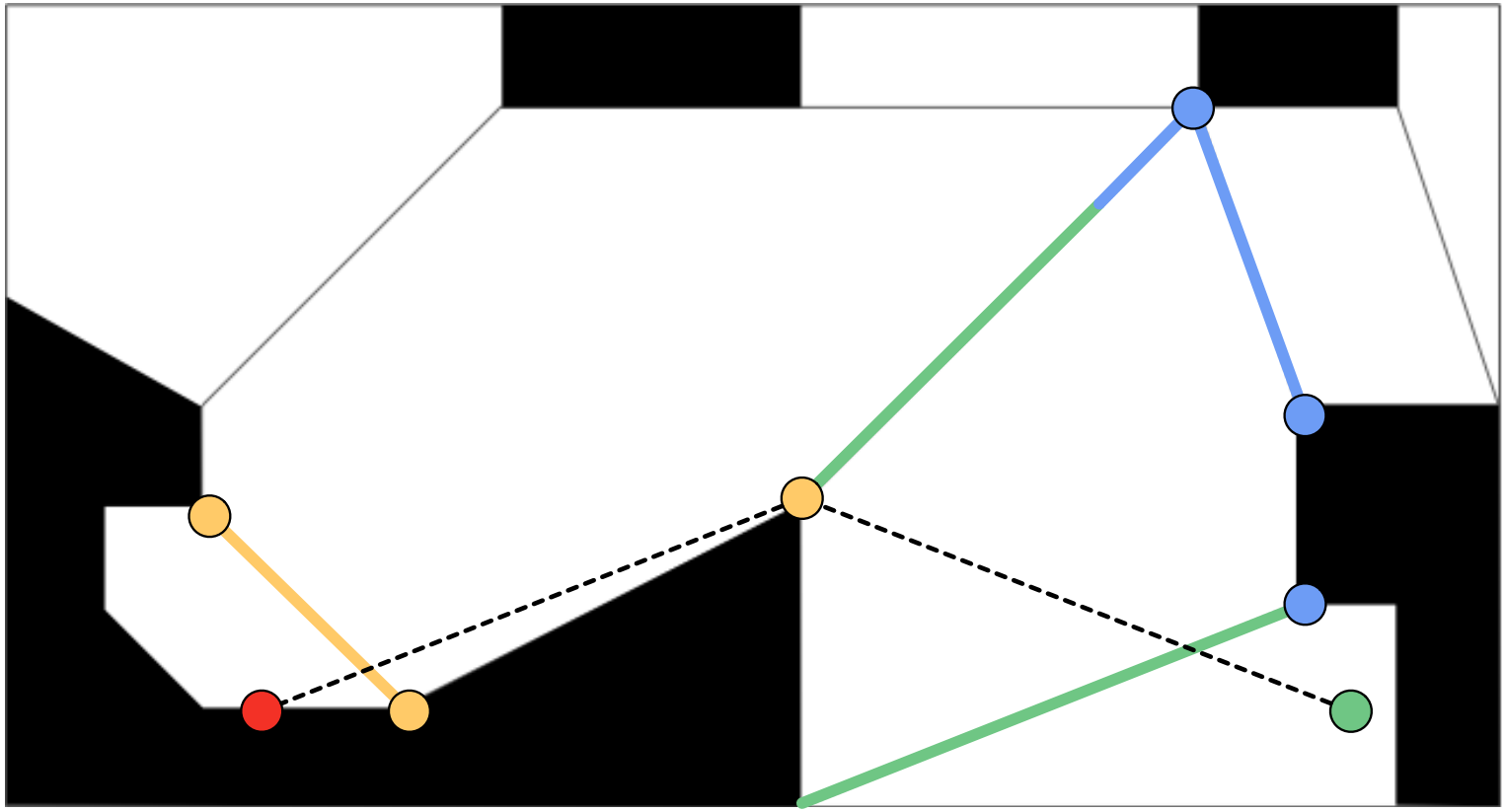
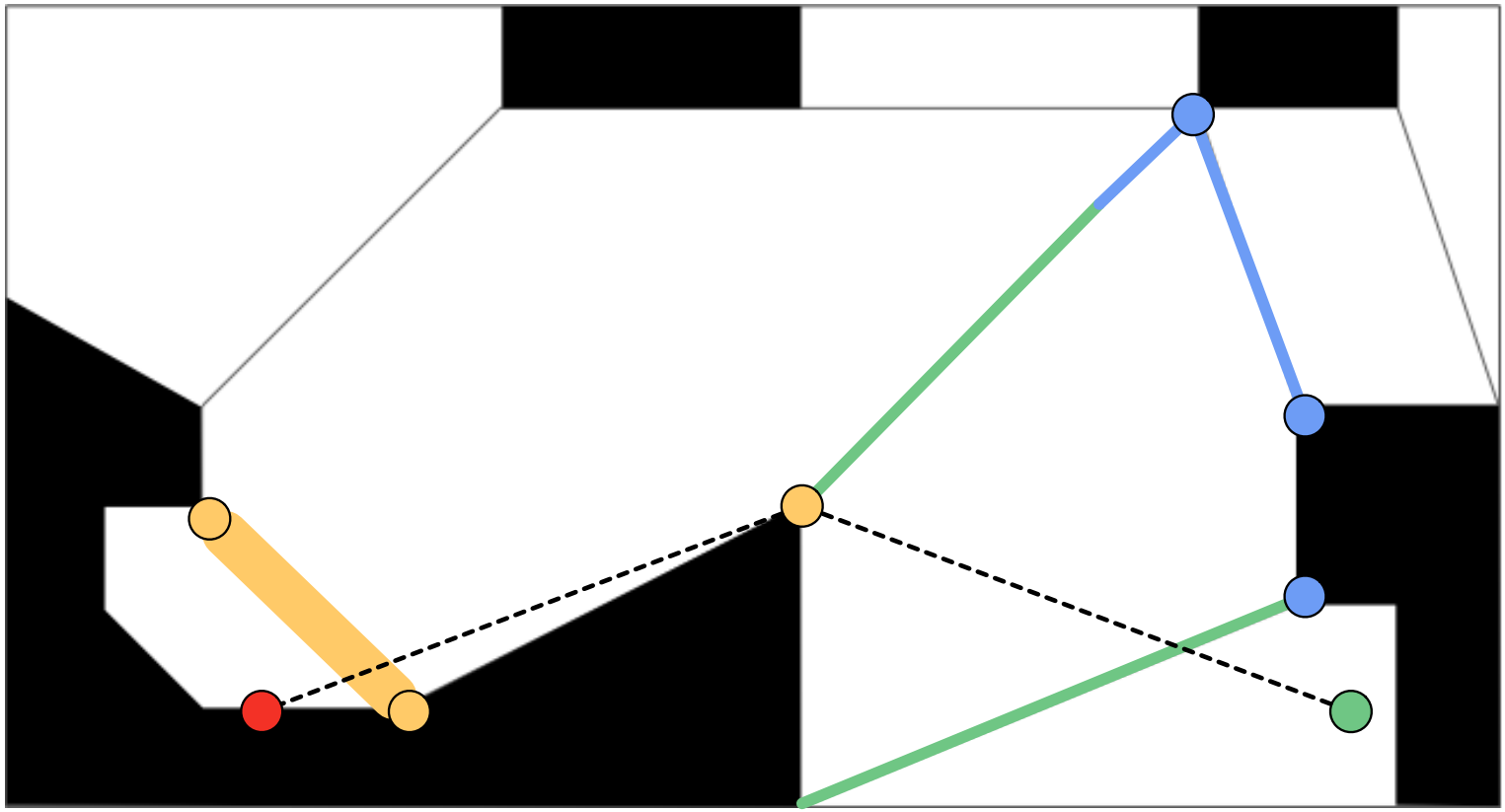


Further optimisations

Intermediate Pruning

Immediately and recursively expand any node that has only a single successor.



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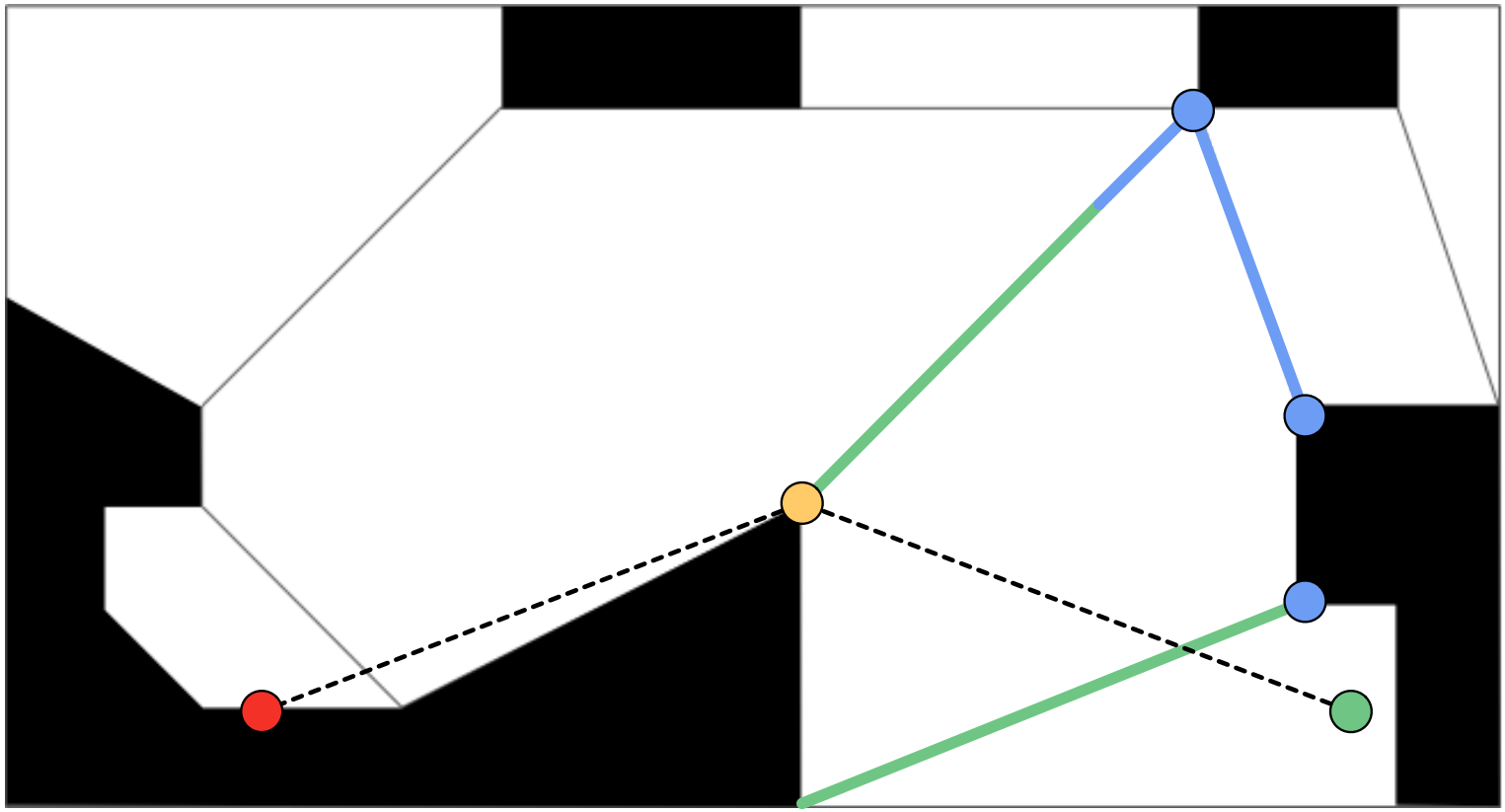
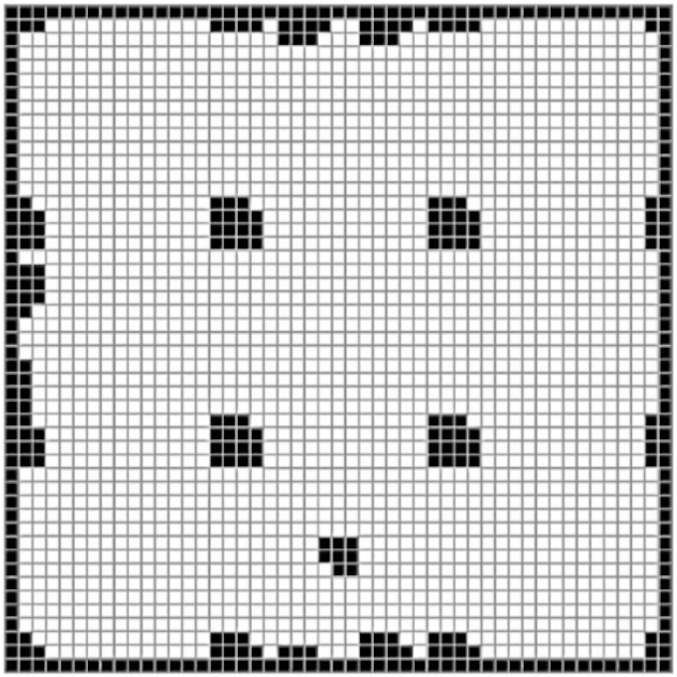


Mesh selection

We generted a variety of meshes including: grids, rectangles, Constrained Delaunay Triangulations (CDT), and greedily merged CDTs.
Rule of thumb: Bigger polys means better performance.



This example grid (ARENA) appears in the game Dragon Age Origins.

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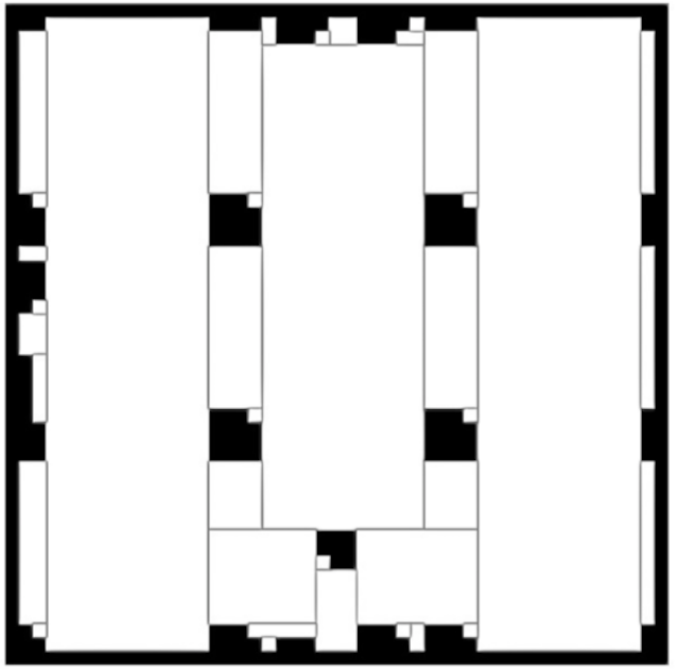

Rectangle mesh, computed by greedily merging grid tiles.

Mesh selection

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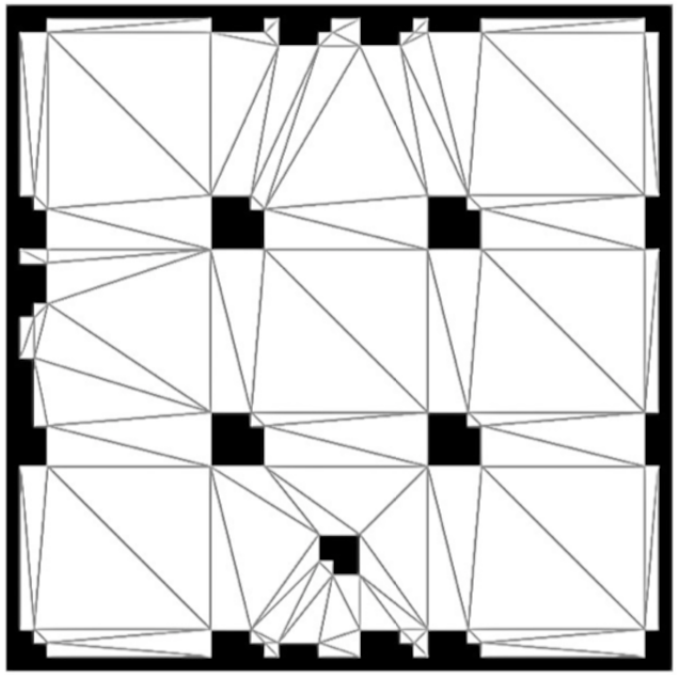

Constrained Delaunay Triangulation, computed with Fade2D

Mesh selection

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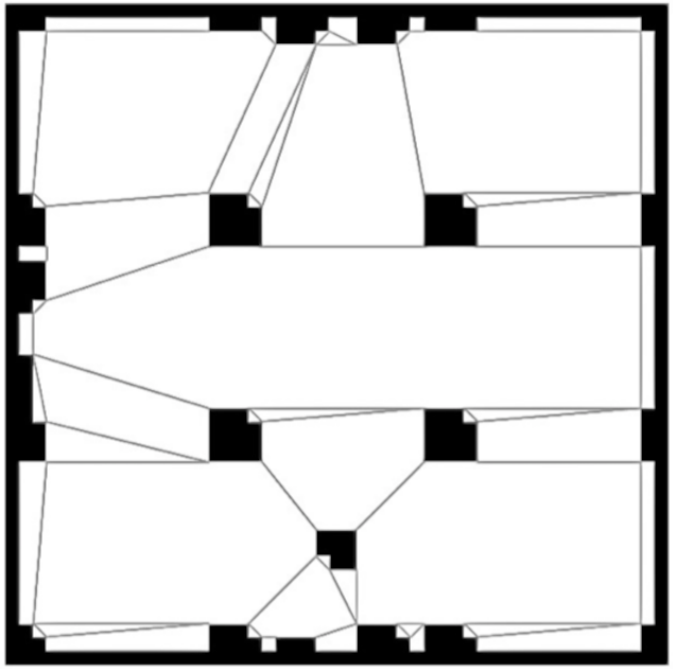


Merged CDT, computed by greedily merging triangles.

Results vs. Anya

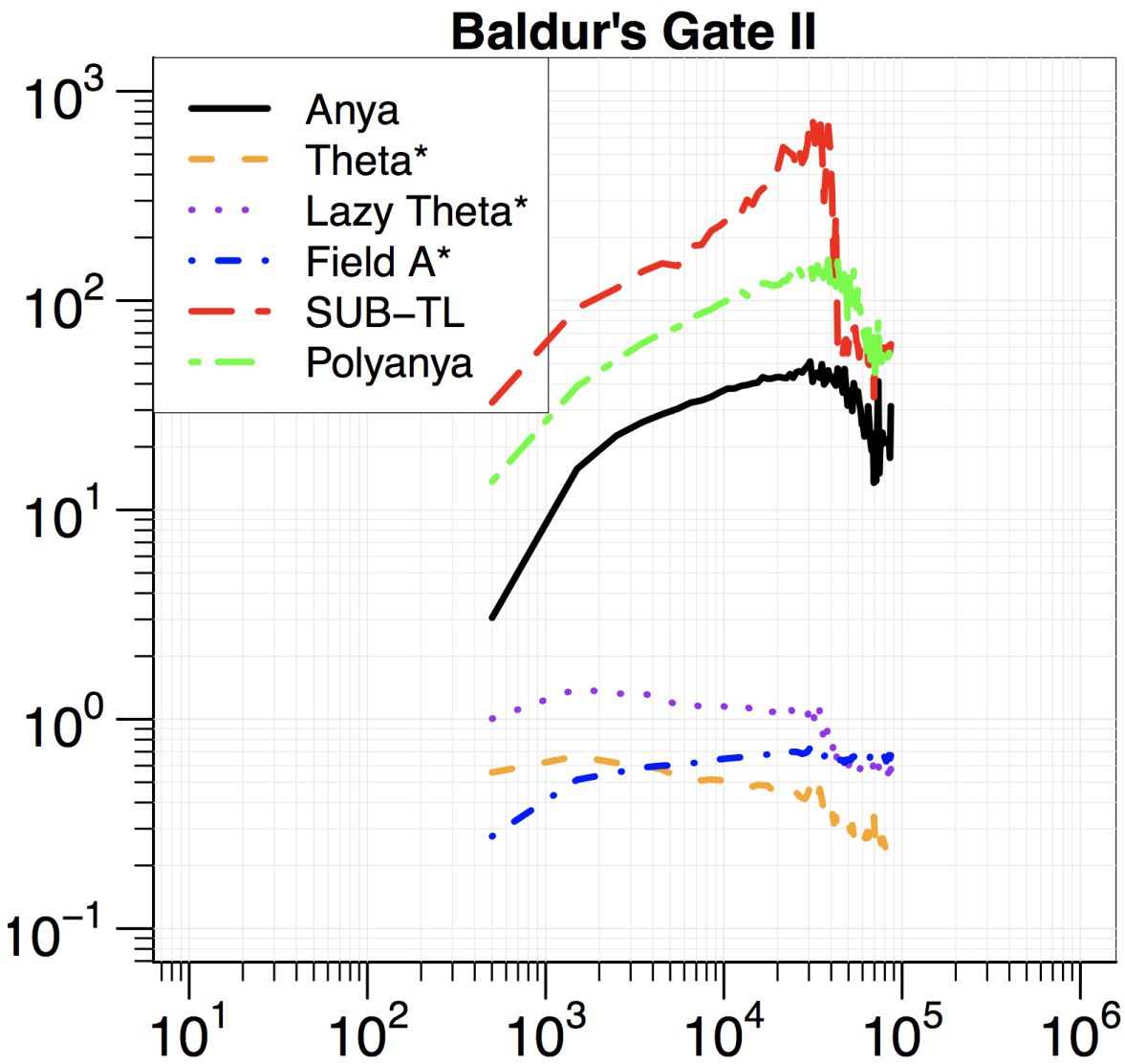
Experiment

Anya on grids vs Polyanya on different mesh types. Performance measured as **node expansions** and **search time speedup** (vs grid A*).

Benchmark	**Average Expansions**				**Average Speedup**			
	Anya	Polyanya			Anya	Polyanya		
		CDT	M-CDT	Rect		CDT	M-CDT	Rect
BG-II	79	67	**37**	131	19.9	40.2	**55.7**	23.7
DA2	228	261	**120**	388	2.5	10.3	**17.6**	7.0
DA:O	956	908	**499**	1725	8.2	8.3	**13.5**	5.3
Mazes	6633	7079	1708	**1614**	22.8	103	**158.2**	157.2
Random	17476	30615	**14243**	17934	0.9	0.6	**1.0**	0.7
Rooms	1431	1862	895	**844**	10	63.8	106.0	**107.8**
SC1	1379	1444	**755**	2537	21	18.1	**29.4**	9.4
WC3	89	109	**55**	132	3.2	30.3	**43.7**	21.0

Table: Highlighted are best results for each metric and benchmark pair.

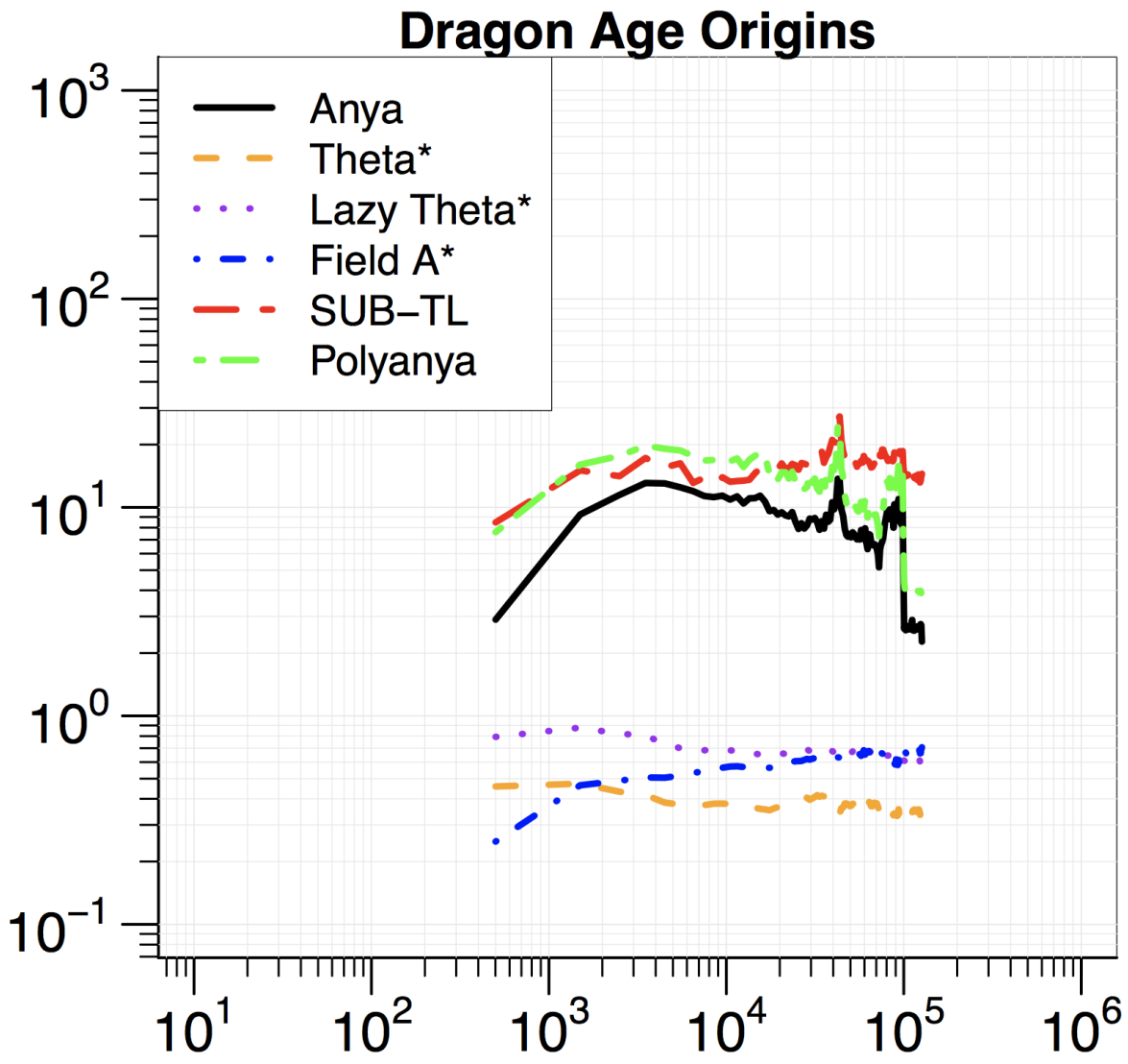
Results on Game Maps



y-axis:
speedup (time) vs grid A*.

x-axis:
problem instances, ordered by difficulty (measured as node expansions required by grid A*).

Results on Game Maps



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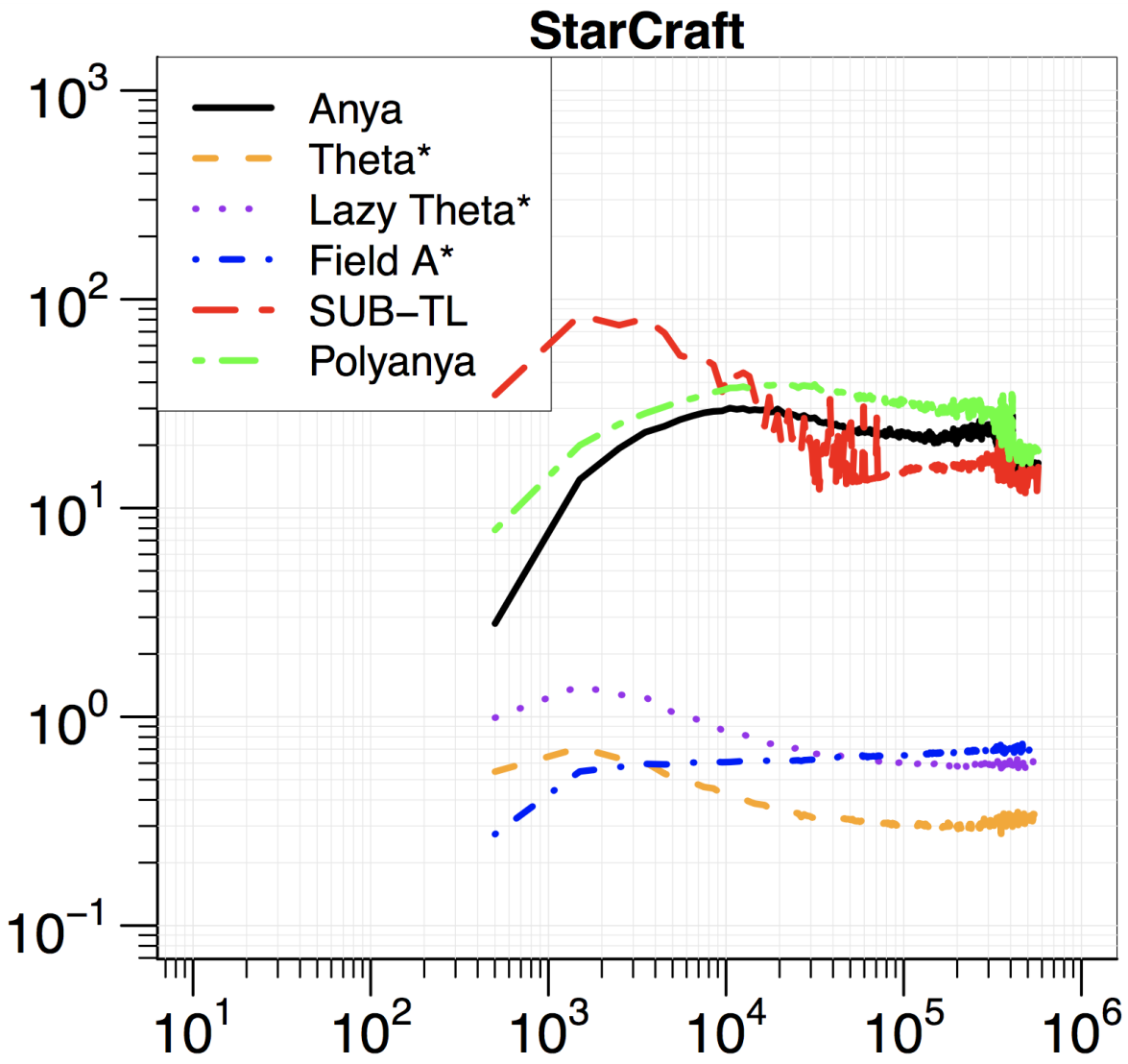

y-axis:
speedup (time) vs grid A*.

x-axis:
problem instances, ordered by difficulty (measured as node expansions required by grid A*).

Perspectives and Future Work

Problems in 2D

Progress in this area

- We have a wide variety of benchmarks
- We understand the characteristics of 2D domains that make search difficult. For example:
  - Grids: Symmetric paths
  - Roads: Highway dimension
- We have sophisticated search algorithms, heuristics and preprocessing techniques
- We can solve 2D problems extremely well and often with very few compromises.

Challenges:

- Maps with dynamically changing costs (up and down)
- Euclidean pathfinding with weighted regions
- Maps with moving obstacles

Problems in 3D

Progress in this area

Some progress has been made:

- We have benchmarks, though only from games [Brewer and Sturtevant, 2018]
- Canonical orderings have been generalised to 3D
- Theta* is generalised to 3D

Challenges

- Orders more states (tens and sometimes hundreds of millions)
- Is 3D search challenging for the same reasons as 2D?
  - We know the continuous 3D problem is intractable in general [Canny and Reif, 1987]
  - But not much else; e.g.
  - Do there exist bounded suboptimal methods analogous to Anya and Polyanya?
- How effective are our existing methods in 3D?

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